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Harmonic Oscillations of Spherical Bodies In a Viscoelastic Environment

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ABSTRACT

In this paper, we consider oscillations of spherical bodies in a deformed medium. The problem reduces to finding those values $\Omega = \Omega_{Rn} + i\Omega_{In}$ *(*Ω_R *- real and* Ω_{In} *- imaginary parts of complex Eigen frequencies) for which the system of equations of motion and the truncated radiation conditions have a nonzero solution to the cash-box of infinitely differentiable functions. It is shown that the problem has a discrete spectrum located on the lower complex plane* $\mathrm{Im}\Omega \le 0$ *and the symmetric spectrum is the imaginary axis.*

Keywords: *oscillations, spherical bodies, medium, frequency, loss of energy.*

INTRODUCTION

In recent years, geophysicists are increasingly attracted to the heterogeneity of the Earth's interior. This interest is explained by the fact that the study of in homogeneities sheds light on the geodynamic processes occurring in the crust and upper mantle, clarifies some problems of geological evolution of the Earth. Thus, the problems associated with the identification of in homogeneities, with the definition of their sizes and physical characteristics, are very important and relevant. Although for these purposes geophysicists use such different approaches as gravity prospecting, electromagnetic methods, study of electrical conductivity, etc., the seismic method is perhaps the most direct and gives the least doubtful results in interpreting.

A comprehensive analysis of the problems of seismic macro defectoscopy is necessary for solving problems of practical importance, such as the investigation of the Earth's core [1], the search for magmatic volcano foci [2], ore bodies [3], etc. Recently, experimental work on the propagation of ultrasonic waves in static models of elastic media containing foreign inclusions and fractured zones has been intensively carried out [4]. The ideal elastic body has no losses [5,6,7,8,9]. Even if the equation is linear with respect to stress and strain, the presence of time derivatives is always associated with dissipation. As a result, with an alternating voltage there is a hysteresis effect. This means that in the frequency range in which attenuation has an appreciable magnitude, the strain will lag behind the voltage. The presence of only a nonlinear connection between stress and deformation (without time derivatives in the equation) causes two effects. Such a connection, firstly, leads to the interaction of the elastic wave under consideration with other waves (for example, with thermal vibrations) and as a result there is a redistribution of energy between the waves. Secondly, the considered wave will generate higher harmonics, transferring their energy to them. In both cases, the interaction depends on the strain amplitude. The nonlinear relationship between stress and strain in the presence of time derivatives also leads to damping, which depends on the strain amplitude. In addition, the study of in homogeneities is of great interest for the study of an important techno physical phenomenon-the behavior of the source of the prepared earthquake. Now among seismologists the concept of a seismic shocks preparation zone is widely accepted, as areas with elastically denser characteristics that change as a result of tectonic movements. From a mechanical point of view, this corresponds to an inhomogeneity with the velocities of the longitudinal and transverse waves that are insignificantly altered with respect to the external elastic medium, and possibly also by the density. Any inhomogeneity, along with its environment, must possess, like any elastic mechanical system, some spectrum of natural frequencies. Since the oscillations of the inclusion and the surrounding medium are interrelated, damping of the oscillations due to the emission of elastic waves takes place and, consequently, the natural frequencies will be complex. Therefore, for practical purposes of identifying possible resonance peaks on the spectral curve and establishing their connection with the corresponding in homogeneities, it is very important to know the natural vibration frequencies of elastic inclusions in an infinite elastic medium [10]. From the physical point of view, the damping in an ideal elastic medium is explained by the radiation of energy excited by the natural vibrations due to divergent elastic waves. The behavior of complex Eigen frequencies depending on the geometric and physic mechanical parameters of the system is investigated. The environment of spherical bodies is considered as elastic, viscoelastic and multicomponent. Interest in the study of the natural frequencies of the elastic inclusion system is also due to the following circumstance. When the inhomogeneity is transmitted through seismic waves or from weak earthquakes or from pulsed artificial sources such as pneumatic emitters, the scattering problem must be solved in a non-stationary formulation. Such a body is characterized by a linear single-valued relationship between stress and strain throughout the entire period of the alternating voltage. Hence it follows that stress and deformation are always in phase. The energy dissipation of an elastic wave will occur if the stress and strain are not connected by an unambiguous dependence during the period of oscillations. The absence of such an unambiguous relationship between stress and deformation arises when temporal derivatives appear in the equation connecting them. As is known, in this case for the calculation of the wave field the stationary solution should be integrated in frequency together with the spectrum of the given incident pulse. Generally speaking, the resulting integral can be calculated by any direct numerical method. In some cases, however, preference should be given to the integration method by using the theory of residues in the form of an expansion in the poles of the integrand, since it is this method that can reveal a number of useful physical features of the diffraction process. We note that the poles of interest to us coincide with the roots of the Eigen frequency equation and, therefore, in order to be able to deal with the problems of non stationary diffraction of elastic waves in the future, we need a careful study of the behavior of the roots of the frequency equations, depending on the ratio of the elastic density parameters of the medium and the inclusion. In this paper we consider oscillations of spherical bodies in a deformed medium [11, 12]. The obtained numerous results are compared on a computer. A piecewise homogeneous mechanical system is regarded as dissipative homogeneous and inhomogeneous.

Investigation of the Mechanisms of Energy Losses in Elastic Spherical Bodies in an Elastic Medium

To explain the **mechanisms of energy loss**, we consider the particular problem of the passage of waves of large length in an elastic medium containing a small volume fraction of hard spherical inclusions. The generalized motion for wave motion in an elastic medium can be written in spherical coordinates for a harmonic periodic wave with an angular frequency ω in the form of $\vec{U} = \vec{U}_0 e^{i\omega t}$ where $\vec{U}_0$ - a function of only spatial coordinates, which can be represented in the form [13]

$$\vec{U}_0 = -\nabla\varphi + \nabla\times[\nabla\times(r\vec{\psi})]$$

Potentials φ and ψ satisfy the equations

$$(\nabla^2+\alpha_c^2)\varphi = 0, \qquad (\nabla^2+\beta_c^2)\vec{\psi} = 0 \qquad (1)$$

where $\alpha_c = \omega\left(\frac{\rho}{\lambda+2M}\right)^{1/2} = \frac{2\pi}{\lambda_e}$,

$$\beta_c = w\left(\frac{\rho}{M}\right)^{1/2} = \frac{2\pi}{\lambda_t},$$

λ_e and λ_t - respectively, the lengths of longitudinal and transverse waves.

The incident longitudinal wave is defined as follows

$$\varphi^{(i)} = \varphi_0 \exp\left[i\omega\left(\frac{x}{\vartheta_l} - t\right)\right], \qquad (2)$$

where φ_0 – amplitude of incident waves; ϑ_l – velocity of propagation of longitudinal waves; ω - frequency. The expression of the reflected wave through the displacement potentials can be written in the form

$$\varphi^{(r)} = \exp(-i\omega t)\sum_{n=0}^{\infty} A_n h_n(\alpha r) P_n(\cos\theta); \qquad (3)$$

$$\psi^{(r)} = \exp(-i\omega t)\sum_{n=0}^{\infty} B_n h_n(\beta r) P_n(\cos\theta);$$

$\alpha^2 = \omega^2 / \vartheta_l^2, \beta^2 = \omega^2 / \vartheta_t^2.$

Here ϑ_t – shear wave velocity; $h_n(\alpha r)$, $h_n(\beta r)$ – spherical Bessel functions; $P_n(\cos\theta)$ – Legendre polynomials, A_n and R_n coefficients. When calculating the Legendre function n>>1 the asymptotic formulas from work [14,15]

$$P_{n-1/2}(\cos\theta) = (2/\pi n \sin\theta)^{1/2}[\cos(n\Delta - \pi/4) + \frac{ctg\theta}{8n}\sin(n\theta - \pi/4) + \theta(1/n^2)].$$

Coefficients A_n and B_n must be determined from the boundary conditions on the surface of the rigid sphere, i.e. from the requirement of continuity of displacements:

$U_r = U(t)\cos\theta$; $U_\theta = U(t)\sin\theta$, (4)

where U(t) – moving the environment. The stress on the surface of the sphere must be related to the equation of its motion as follows:

$$\rho_0 \pi a^3 \frac{d^2U}{dt^2} = 1{,}5\pi a^2 \int_0^\pi (\sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta)\sin\theta\, d\theta, \quad (5)$$

Where a - is the radius and ρ_n – the density of a spherical switch-off. The movement of the sphere of the problem under consideration must be a harmonious one:

$$U(t) = Ue^{iwt}.$$

Under the condition of long wavelengths (1), (2) and (3), the acting external force is expressed as follows:

$$F(w) = -\frac{\rho_m V_0}{\tau_0^2}\left\{\frac{9[V(w)-U(w)]}{2x^2+1}\left(1 - ia\frac{2x^3+1}{2x^2+1}\right) - a^2 f_1 V(w) + a^2 f_2 V(w)\right\},$$

where ρ_m – density surrounding a particle of an elastic medium; V_0 – inclusion volume; $V(\xi)$– switching motion; ξ - some constant number

$$f_1 = \frac{2 + 9x + x^2 + [18x^3(x+2)]/(2x^2+1)^2 - Q}{2x^2+1};$$

$$f_2 = \frac{4{,}5 + 9x + 3x^2 + 18x^3(x+2)/(2x^2+1)^2}{2x^2+1};$$

$$Q = 9x(x+1)(2x+1)/(2x^2+1);$$

$$x = \beta/\alpha = \vartheta_l / \vartheta_t;$$

$$\tau_0 = \alpha / \vartheta_l.$$

Now let us turn to the case $\rho_b >> \rho_m$, those. When the density of the inclusion material is much greater than the density of the surrounding medium. Under this condition, the last two terms can be neglected

$$F(w) = -\frac{9\rho_m V_0}{\tau_0^2} - \frac{V(w)-U(w)}{2x^2+1}\left(1 - ia\frac{2x^3+1}{2x^2+1}\right). \quad (6)$$

Substituting (6) into the equation of motion (5) and writing the result in terms of time derivatives, we obtain

$$\rho_0\frac{d^2V}{dt^2} + \frac{9\rho_m(2x^3+1)}{\tau_0(2x^2+1)^2}\left(\frac{dV}{dt} - \frac{dU}{dt}\right) + \frac{9\rho_m}{\tau_0^2(2x^2+1)}(V-U) = 0. \quad (7)$$

The last term in (7) characterizes the elastic energy, similar to the spring energy, and the velocity terms describe the phenomenon of energy dissipation because of the scattering of wave energy.

$$\eta = \frac{2\rho_m(2x^3+1)}{2\rho_b\tau_0(2x^2+1)}; \quad \omega^2 = \frac{2\rho_m}{2\rho_b\tau_0(2x^2+1)}. \quad (8)$$

The first term in (8) characterizes the attenuation coefficient, and the second term describes the natural frequencies. To determine the damping in the first place, it is necessary to determine the scattering coefficient. Scattering is the ratio of the total energy considered per unit time to the energy transported by the incident wave per unit time through a unit area perpendicular to the propagation direction [13]. Then the expression for the scattering cross section is given by the formulas, the expression for the energy dissipation rate can be represented in the form:

$$F_S = \frac{iw\pi a^2}{2}\int_0^\pi \begin{bmatrix} (\sigma_{rr}U_{rr}^* + \sigma_{r0}U_0^* + \sigma_{rr}U_\varphi^*) - \\ (\sigma_w^* U_r + \sigma_\alpha^* U_0 + \sigma_\varphi^* U_\varphi) \end{bmatrix} \sin\theta\, dt,$$

where $\sigma_{rr}, \sigma_{r\theta}, \sigma_{\theta\theta}, U_r, U_\varphi$ - respectively, the components of the stress and displacement tensor. Attenuation for scattering by mutually independent centers is

$$\alpha_\partial = \frac{1}{2} n_0 \gamma,$$

where n_0 - concentration of scattering centers; γ -scattering for a single scattering center. Let us consider an infinite isotropic elastic medium in which an elastic medium is enclosed, from a material different from the surrounding medium. The environment and the sphere are in a state of periodic motion, and far from the sphere. This motion is the propagation of a monochromatic flat longitudinal wave

$$lxp\left[i(\omega t - \vec{k}\,\vec{r})\right].$$

At any point in the medium, the motion is assumed to consist of the sum of the fields of this plane longitudinal wave and the field of spherical waves due to the presence of a sphere.

These wave fields will be called respectively incident and scattered waves.

When considering the motion of an elastic medium, one can speak of mechanical energy, which is carried by a propagating wave. The scattering coefficient expresses the relationship between the energies of the incident and scattered waves in a sphere containing the scattering center. Thus, the scattering coefficient is expressed in terms of the characteristics of the incident and scattered waves. The incident wave is given by the conditions of motion and the corresponding boundary conditions. The equation of motion for the displacement U in an isotropic elastic medium has the form (1). A wave in the environment, as already

It was noted that the sum of the incident and scattered waves:

$$U_0 = U_i + U_s \qquad (9)$$

Where the subscripts refer respectively to the incident and scattered waves. Equations (1) and (9) are linear, so for a region outside the sphere

$$\varphi_1 = \varphi_I + \varphi_S ; \qquad \psi_I = \varphi_I + \varphi_S = \psi_S \qquad (10)$$

$$(\nabla^2 + K_1^2)\varphi_i = 0, \quad (\nabla^2 + K_1^2)\varphi_s = 0, \quad (\nabla^2 + H_1^2)\psi_s = 0,$$

where the subscript 1 refers to the medium: ψ_i is zero, since a purely longitudinal wave is considered. With in the scheme

$$\varphi_2 = \varphi_q , \qquad \psi_2 = \psi_q \qquad (11)$$

$$(\nabla^2 + K_2^2)\varphi_q = 0, \quad (\nabla^2 + H_2^2)\psi_q = 0,$$

index 2 refers to the material from which the sphere consists, and the index q- to the wave inside the sphere of expression differ by the factor $(-i)^{m+1} a(2m+1)$ from the expressions used in (11). The multiplier is introduced in order that the coefficients A_m, B_m, C_m, D_m were dimensionless, and also for some simplification of the subsequent equations.

When choosing the form, it was taken into account that for large r the solution should be of the form of a simple harmonic and spherical wave. Let us write the incident wave in the form

$$U_i = e^{-iR_1Z} Z_1, \qquad (12)$$

Where, Z_1 -единичный вектор в направлении распространения и Z=r cos θ

Given that, $U = -\nabla\varphi_i$, can express φ_i in the shape of

$$\varphi_i = (K_1)^{-1} \sum_{m=0}^{\infty} (-i)^{m+1} (2m+1) J_m(K_1 r) P_m(\cos\theta)$$

As a result, a wave outside the sphere is represented as follows:

$$\varphi = \psi_i + \psi_s \qquad \psi = \psi_s$$

$$\varphi_i = \sum_{m=0}^{\infty} (-i)^{m+1} (K_1)^{-1} (2m+1) J_m(K_1 r) P_m \cos\theta \qquad (13)$$

$$\varphi_s = \sum_{m=0}^{\infty} (-i)^{m+1} a(2m+1) A_m h_m (K_1 r) P_m \cos\theta$$

$$\psi_s = \sum_{m=0}^{\infty} (-i)^{m+1} a(2m+1) B_m h_m (H_1 r) P_m(\cos\theta),$$

but for a wave inside the sphere $\varphi = \varphi_q$, $\psi = \psi_q$;

$$\varphi_q = \sum_{m=0}^{\infty} (-i)^{m+1} a(2m+1) C_m J_m(K_2 r) P_m(\cos\theta), \qquad (14)$$

$$\psi_q = \sum_{m=0}^{\infty} (-i)^{m+1} a(2m+1) D_m j_m(H_2 r) P_m(\cos\theta)$$

Coefficients A_m, B_m, C_m and D_m must be determined from the boundary conditions (4). Expression (13) and (14) can be used by calculating the displacements and voltages from the potentials ψ and π.

From the equation, bearing in mind that φ and ψ does not depend on φ, we get

$$U_r = \frac{\partial\varphi}{\partial r} - \frac{1}{r}\Omega\psi, \quad U_\theta = -\frac{1}{r}\frac{\partial\varphi}{\partial\theta} + \frac{1}{r}\frac{\partial^a}{\partial\theta\partial r}(r\psi), \quad U_\psi = 0$$

Here Ω- operator, which has the following form

$$\frac{1}{Sin\beta\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}).$$

The stresses in an isotropic elastic medium in Cartesian coordinates are equal to

$$\sigma_{ij} = \lambda(\nabla\cdot S)\delta_{ij} + \mu(\frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i}), \qquad (15)$$

Where, $\delta_{ij} = 1$ at i=j and $\delta_{ij} = 0$ at $i \neq j$. Passing to spherical coordinates and using (15), we obtain

$$\sigma_{rr} = \rho w^{-2}\left\{\varphi + \frac{2}{H^2}[\frac{2}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\Omega\varphi - \frac{\partial}{\partial r}(\frac{1}{r}\Omega\psi)\right\} \qquad (16)$$

$$\sigma_{r\theta} = -2\frac{\rho w^2}{v^2\partial\theta}\left[\frac{1}{r}\frac{2\varphi}{\partial r} - \frac{1}{r^2}\varphi + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}(1 + \frac{H^2r^2}{2})\psi + \frac{1}{r^2}\Omega\psi\right], \sigma_{\varphi r} = 0$$

For the scattered wave, φ and ψ are expressed by equations (6) and (16). Substituting these expressions in (14) and using (16), we obtain that the energy flux of the scattered wave through a spherical surface with a radius greater than the radius of the scattering center

$$F_s = 2\pi 01 w^2 a^2 \sum_{m=0}^{\infty}(2m+1)\left[\frac{1}{k_1}|Am|^2 + \frac{m(m+1)}{H1}|Bm|^2\right]\cdot$$

From (8) for the scattering cross section we obtain

$$\gamma = 4\pi a^2 \sum_{m=0}^{\infty}(2m+1)\left[|Am|^2 + m(m+1)\right]\frac{K_1}{H_1}|Bm|^2]\cdot$$

Since the A_m and B_m dimensionless quantities, the scattering coefficient has the dimension of the area, as it should be, by definition. The dimensionless scattering coefficient has the following form

$$\gamma_N = \frac{\gamma}{\pi a^2} \quad . \qquad (17)$$

In this way, γ_N is defined as the ratio of the total energy flux in the scattered wave to the energy flux in the incident wave and has the following form

$$\gamma_N = 4\sum_{m=0}^{\infty}(2m+1)\left[|A_m|^2 + m(m+1)\frac{K_1}{H_1}|B_m|^2\right]$$

Below is a calculation of this value for two cases: an isotropic elastic sphere and a spherical cavity in an elastic medium.

Elastic Sphere

The boundary conditions are continuity of stresses and displacements on the boundary, ie the following relations are satisfied:

$$U_{ri} + U_{rs} = U_{rq}; \quad U_{\theta i} + U_{\theta s} = U_{\theta q}; \qquad (18)$$

$$\sigma_{rri} + \sigma_{rrs} = \sigma_{rrq}; \quad U_{\theta ri} + U_{\theta rs} = U_{\theta rq}.$$

The use of these relations and equations (13), (14), (18) gives the following equations, linear with respect to A_{em}, B_{em}, C_{em} and D_{em}:

$$A_{em}(K_1 a)h_m(K_1 a) + B_{em}M(H_1 a)h_m(Ha) - C_{em}(K_2 a)J_m(K_2 a) -$$
$$- D_{em}M(H_2 a)J_m(H_2 a) = (-1)^m J_m(k,a).$$

$$A_{em}h_m(K_1 a) - B_{em}[(m+1)h_m(H_1 a) - (H_1 a) - (H_1 a)h_m(H_1 a)] - C_{em}J_m(k_2 a) +$$
$$+ D_{em}[(m+1)J_m(H_2 a) - H_2 a J_m(H_2 a)] = (-1)^m (k_1 a)^{-1} J_m(k_1 a)$$

$$A_{em}\lfloor (H_1 a)^2 h_m(k_1 a) - 2(m+2)k_1 a h_{m+1}(k_1 a)\rfloor +$$
$$+ B_{em}\left[(H_1 a)^2 h_m(H_1 a) - 2(m+2)H_1 a h_{m+1}(H_1 a)\right] -$$
$$- C_{em}\left(\frac{\mu_2}{\mu_1}\right)\left[(H_2 a)^2 J_m(k_2 a) - 2(m+2)k_2 a J_{m+1}(k_2 a)\right] - D_{em}\left(\frac{\mu_2}{\mu_1}\right)m\cdot$$
$$\cdot\left[(H_2 a)^2 J_m(H_2 a) - 2(m+2)H_2 a J_{m+1}(H_2 a)\right] = (-1)^m (k,a)^{-1}$$
$$\left[(H_1 a)^2 J_m(k_1 a) - 2(m+2)(k_1 a)J_{m+1}(k_1 a)\right]; \qquad (19)$$

$$A_{em}[(m-1)h_m(k_1 a) - k_1 a h_{m+1}(k_1 a)] -$$
$$- B_{em}\left[(m^2 - 1 - \frac{1}{2}H_1^2 a^2)h_m(H_1 a) + (H_1 a)h_{m+1}(H_1 a)\right] -$$
$$- C_{em}\left(\frac{\mu_2}{\mu_1}\right)\cdot[(m-1)J_m(k_2 a) - (k_2 a)J_{m+1}(k_2 a)] + D_{em}\frac{\mu_2}{\mu_1}\cdot$$
$$\cdot\left[(m^2 - 1 - \frac{J}{2}H_2^2 a^2)J_m(H_2 a)J_{m+1}(H_2 a)\right]$$
$$= (-1)^m (K_1 a)^{-1}[(m-1)J_m(k_1 a) - (k_1 a)J_{m+1}(k_1 a)].$$

The index e denotes coefficients related to the elastic sphere Setting m = 0, we find from (19) that

$$A_{io} = \frac{1}{k_1 a}\frac{\left[(H_1 a)^2 - (4+E)\right]\sin K_1 a + (4+E)k_1 a\cos k_1 a}{\left\{\left[H_1 a^2 - (4+E)\right]^2 + (4+E)^2 (k_1 a)^2\right\}^{1/2}} \qquad \exp\left\{i\left[k_1 a - arctg\frac{(H_1 a)^2 - (4+E)}{(4+E)k_1 a}\right]\right\},$$

where

$$E = \frac{\rho_2}{\rho_1}\left[\frac{(\chi - a)^2}{1 - k_2 arctg k_2 a} - 4\left(\frac{\chi_1}{\chi_2}\right)^2\right] \qquad (20)$$

For m> 0, the exact analytic solution of the system of equations (19) turns out to be rather complicated, and it is practically better to solve it numerically. Equations (19) are generally complex and equivalent to the system of eight real linear equations. Introduction the notation

$$A_{em} = X_1^{(m)} + ix_2^{(m)};$$

$$B_{em} = X_3^{(m)} + ix_4^{(m)};$$

$$C_{em} = X_5^{(m)} + ix_6^{(m)};$$

$$D_{em} = X_7^{(m)} + ix_8^{(m)};$$

where $x^{(m)}$- real values, it is possible to rewrite (19) in the following form:

$$\sum_{j=1}^{\infty} \varepsilon_{ij}^{(m)} x_j^{(m)} = \delta_i^{(m)}, \quad i=1,2,\ldots,8, \qquad (21)$$

Where, the actual matrix elements ε_{ij} and δ_i have the form

$$\varepsilon_{11}^{(m)} = -k_2 a j_{m+1}(k_2 a), \qquad \varepsilon_{12}^{(m)} = k_1 a H_{m+1}(k_1 a),$$

$$\varepsilon_{13}^{(m)} = m H_1 a j_{m+1}(H_1 a), \qquad \varepsilon_{14}^{(m)} = m H_1 a h_{m+1}(H_1 a),$$

$$\varepsilon_{15}^{(m)} = -k_2 d j_{m+1}(k_2 a), \qquad \varepsilon_{16}^{(m)} = 0$$

$$\varepsilon_{17}^{(m)} = -m H_2 d j_{m+1}(H_2 a), \quad \varepsilon_{18}^{(m)} = 0$$

$$\varepsilon_{31}^{(m)} = j_m(k_1 a), \quad \varepsilon_{32}^{(m)} = H_m(k_1 a),$$

$$\varepsilon_{33}^{(m)} = -[(m+1) J_m(H_1 a) - H_1 a J_{m+1}(H_1 a)],$$

$$\varepsilon_{34}^{(m)} = [(m+1) J_m(H_1 a) - H_1 a J_{m+1}(H_1 a)],$$

$$\varepsilon_{35}^{(m)} = -j_m(k_2 a), \qquad \varepsilon_{36}^{(m)} = 0$$

$$\varepsilon_{37}^{(m)} = (m+1) J_m(H_2 a) - H_2 a j_{m+1}(H_2 a), \qquad \varepsilon_{38}^{(m)} = 0$$

$$\varepsilon_{51}^{(m)} = (H_2 a)^2 j_m(k_1 a) - 2(m+2) k_1 a j_{m+1}(k_1 a),$$

$$\varepsilon_{52}^{(m)} = (H_1 a)^2 h_m(k_1 a) - 2(m+2) k_1 a h_{m+1}(k_1 a)$$

$$\varepsilon_{53}^{(m)} = m[(H_1 a)^2 j_m(H_1 a) - 2(m+2) H_1 a j_{m+1}(H_1 a)],$$

$$\varepsilon_{54}^{(m)} = m[(H_1 a)^2 h_m(H_1 a) - 2(m+2) H_1 a h_{m+1}(H_1 a)],$$

$$\varepsilon_{57}^{(m)} = -\left(\frac{\mu_2}{\mu_1}\right)[(H_2 a)^2 J_m(H_2 a) - 2(m+2) H_2 a J_{m+1}(H_2 a)], \varepsilon_{58}^{(m)} = 0;$$

$$\varepsilon_{71}^{(m)} = (m-1) J_m(K_1 \theta) - K_1 a J_{m+1}(K_1 a);$$

$$\varepsilon_{72}^{(m)} = (m-1) J_m(K_1 \theta) - K_1 a h_{m+1}(K_1 a);$$

$$\varepsilon_{73}^{(m)} = -\left[(m^2 - 1 - \frac{1}{2} H_1^2 a^2) J_m(H_1 a) + H_1 a J_{m+1}(H_1 a)\right];$$

$$\varepsilon_{74}^{(m)} = -\left[(m^2 - 1 - \frac{1}{2} H_1^2 a^2) H_m(H_1 a) + H_1 a H_{m+1}(H_1 a)\right];$$

$$\varepsilon_{75}^{(m)} = -\left(\frac{\mu_2}{\mu_1}\right)[(m-1) J_m(K_2 a) - K_2 a J_{m+1}(K_2 a)], \qquad \varepsilon_{76}^{(m)} = 0;$$

$$\varepsilon_{2n,l}^{(m)} = \begin{cases} -\varepsilon_{2n-1,\ l+1}^{(m)} & l-\text{odd} \\ -\varepsilon_{2n-1,\ l+1}^{(m)} & l-\text{even} \end{cases}$$

$$\delta_i^{(m)} = \begin{cases} (-1)^m (k_1 a)^{-1} \varepsilon_{i,1,}^{(m)} & l-odd \\ 0, & l-\text{even} \end{cases}$$

When $la \ll 1$ we can obtain an approximate expression for the scattering coefficient, neglecting terms of more than second order in $\kappa_1 a$ in the expansion in a series of functions of Bessel and Hankel. Then

$$\gamma_N \approx \frac{4}{9} g e (k_1 a)^4, \qquad (22)$$

Where,

$$g_e = \left\{\frac{3(H_1/K_1)^2}{[3(H_2/K_2)^2 - 4](\mu_2/\mu_1) + 4} - 1\right\}^2 + \frac{1}{3}\left[1 + 2\left(\frac{H_1}{K_1}\right)^3\right]\left[\left(\frac{H_2}{H_1}\right)^2 \frac{\mu_2}{\mu_1} - 1\right]^2 +$$

$$+ 40\left[2 + 3\left(\frac{H_1}{K_1}\right)^5\right]\left\{\frac{(\mu_2/\mu_1) - 1}{2[3(H_1/K_1)^2 + 2]\frac{\mu_2}{\mu_1} + 9(H_1/K_1)^2 - 4}\right\}$$

Expression (22) is an approximation of the Rayleigh formula.

Spherical cavity. In this case there is no wave inside the sphere, so that ψ_q and $\pi_q = 0$. Therefore, it is necessary to determine only two groups of coefficients $\{A_m\}$ and $\{B_m\}$, i.e. only two boundary conditions are necessary.

We require that the stress components be continuous at the boundary for r=a

$$\sigma_{rri} + \sigma_{rrs} = 0, \quad \sigma_{\theta ri} + \sigma_{\theta rs} = 0.$$

These conditions lead to the following equations for A_m and B_m:

$$A_{cm}[(m-1) h_m(K_1 a) - K_1 a h_{m+1}(K_1 a)] + B_{cm}$$

$$\left\{\left[\frac{1}{2}(H_1 a)^2 - (m^2 - 1)\right] h_m(H_1 a) - H_1 a h_{m+1}(H_1 a)\right\} = (-1)^m (K_1 a)^{-1}.$$

$$[(m-1) J_m(K_1 a) - K_1 a J_{m+1}(K_1 a)]$$

The index denotes coefficients related to the cavity. Hence we obtain

$$A_{cm} = (-1)^m \frac{1}{H_1 a} \frac{A_{cm}}{D_{cm}},$$

$$B_{cm} = \frac{(-1)^m 2(m-1)(m+2) - (H_1 a)^2}{(K_1 a)^2} \frac{1}{\Delta_{cm}},$$

$$\left\{\begin{matrix} A_{cm} \\ \Delta_{cm} \end{matrix}\right\} = (H_1 a)^2 \left[-(m-1)(2m+1) + \frac{1}{2}(H_1 a)^2 \left\{\begin{matrix} J_m(K_q a) \\ h_m(K_1 a) \end{matrix}\right\}\right]$$

$$h_m(H_1 a) + H_1 a[2m(m-1)(m+2) - (H_1 a)^2] - \left\{\begin{matrix} J_m K_1 a \\ h_m K_1 a \end{matrix}\right\} h_{m+1} \cdot$$

$$\cdot (H_1 a) + 2K_1 a[(m^2 - 1)(m+2) - (H_1 a)^2]\left\{\begin{matrix} J_{m+1}(K_1 a) \\ h_{m+1}(K_1 a) \end{matrix}\right\} h_m(H_1 a) -$$

$$-2(K_1 a)(H_1 a) \cdot (m-1)(m+2)\left\{\begin{matrix} J_{m+1}(K_1 a) \\ h_{m+1}(K_1 a) \end{matrix}\right\} h_{m+1} 1(H_1 a)$$

Expressions for A_{co} has the form

$$A_{co} = \frac{1}{K_1 a} \frac{\left[(H_1 a)^2 - 4\right] \sin K_1 a + 4K_1 a \cos K_1 a}{\left\{\left[(H_1 a)^2 - 4\right]^2 + 16(K_1 a)^2\right\}^{1/2}} \cdot$$

$$\cdot 12p\left[i(K_1 a - arctg\frac{(\chi_1 a)^2 - 4}{4K_1 a})\right]$$

From a comparison of this expression with (20) it follows that the case of a spherical cavity can be obtained from the case of an elastic sphere, (22) $E \to 0$.

Neglecting the terms in the quadratic l_a in the expansion of spherical Bessel functions, we arrive at the Rayleigh approximation for $la << 1$

$$\gamma_N = \frac{4}{9} g_c (K_1 a)^4$$

Where

$$g_c = \frac{4}{3} + 40\frac{2 + 3(\chi_1 / K_1)^5}{\left[4 - 9(H_1 / K_1)^2\right]^2} - \frac{3}{2}(\frac{\chi_1}{K_1})^2 + \frac{2}{3}(\frac{\chi_1}{K_1})^3 + \frac{2}{16}\cdot(\frac{H_1}{K_1})^4 ,$$

$$K = \frac{\omega}{v_e} = \frac{\omega}{\left[(\lambda + 2\mu)/\rho\right]^{1/2}}, \qquad \chi = \frac{\omega}{v_t} = \frac{\omega}{(\mu/\rho)^{1/2}} ,$$

λ and μ – Lame coefficients; G and μ - the shear modulus and $\frac{\chi}{K} = \frac{v_i}{v_t}$; $\frac{\chi_1}{\chi_2} = \frac{vt_2^1}{vt_1^2}$

It is easy to see that both for the sphere and for the cavity δ is expressed only in terms of the velocities of the longitudinal and transverse waves. The expression for g_i through a bulk module $K = \lambda + 2/3\mu$

$$g_c = [(1 + \frac{4}{3}\frac{G_1}{K_2})^2]^{-1}[(\frac{K_2}{K_1})^2]^{-1}(\frac{K_2 - K_1}{K_1})^2 + \frac{1}{3}[1 + 2(\frac{K_1}{G_1} + \frac{4}{3})^{3/2}]$$

$$(\frac{\rho_2 - \rho_1}{\rho_1})^2 + 40[2 + 3(\frac{K_1}{G_1} + \frac{4}{3})^{5/2}] \times \left\{[3(2\frac{G_2}{G_1} + 1)\frac{K_1}{G_1} + 4(3\frac{G_2}{G_1} + 2)]^2\right\}^{-1} \cdot (\frac{G_2 - G_1}{G_1})^2$$

In the Rayleigh approximation $la << 1$ The scattering coefficient for transverse waves in the case of an elastic sphere is given by the formula

$$\gamma_N = \frac{8}{3}[1 + \frac{1}{2}\frac{K_1^3}{H_1^3}][3\frac{H_1^2}{H_2^2} - 3\frac{H_2^2}{H_1^2} - 4\frac{K_2^2 H_1^2}{H_2^4} + 10\frac{K_2^2}{H_2^2} - 6\frac{K_2^2}{H_1^2}]$$

$$[1 - 10\frac{H_1}{K_1} + 6\frac{K_2}{K_1} - 6\frac{K_2}{K_1}\frac{H_2^2}{H_1^2} + 9\frac{H_2^2}{H_1^2}]^2 \cdot (H_1 a)^4$$

as for longitudinal waves, scattering is proportional to the fourth power of the frequency, an analogous frequency dependence in the Rayleigh approximation is obtained also in the scattering of transverse waves on the cavity.

SUMMATION OF SERIES

When calculating the scattering coefficient from formula (14), the summation is carried out until the ratio of the current term to the current partial sum becomes less 10^{-10}, The convergence of the series (14) is given by numerical experiments.

In calculating the scattering cross sections for the elastic sphere and for the spherical cavity, according to Eq. (22), the Roel approximation was also calculate. For $\kappa a << 1$ expression (22) should give the same result as these approximations.

Table1. *Error in determining the frequency and damping coefficients for different numbers of rows.*

Inclusions in the environment	Ω	η	Ошибка, %	Число членов
Germanium in aluminum	1.0	0,46321	1	4
	5.0	2,86653	0,5	11
	10.0	3,51241	0,5	16
Aluminum in germanium	1.0	0,21235	1,8	4
	5.0	1,24673	0,7	11
	10.0	2,323573	0,5	16

For complex roots, Mueller's method of I. Barstow simplifies calculations and provides faster convergence than Newton's method and Barstow simplifies calculations and provides more if the roots are close to each other. Table 1 shows examples of errors determined by the formula and the necessary numbers of terms in the series. It is seen that to calculate the damping factor and the natural frequencies it is necessary to take 11-16 terms of the series. In this case, the rounding error is 1% ($\rho_m / \rho_0 = 0{,}02$; $\overline{C} = 0{,}5$; a=1; 2. $\rho_m / \rho_0 = 50$; $\overline{C} = 0{,}5$; a=1).

The main conclusions of the paper are as follows:

- A theory and methods for calculating the complex natural frequencies of oscillations of an elastic spherical inhomogeneity in an elastic medium are constructed. The formulation of the problem is proposed for the natural oscillations of cylindrical bodies in a deformed medium. The task is to find those $\Omega = \Omega_R + i\Omega_i$ (Ω_R - real and Ω_i - imaginary parts of complex Eigen frequencies) for which the system of equations of motion and the truncated radiation conditions have a nonzero solution in the class of infinitely differentiable functions. It is shown that the problem has a discrete spectrum.
- Detailed numerical calculations of natural frequencies and Q-factors for the radial and

first three vibration numbers of the torsion and spheroidal classes are performed. The case was considered when the elastic density characteristics of the inclusion and the host medium differ not too strongly.

- The differential and total scattering cross sections for various low-contrast inclusions are calculated. It is shown that in the region close to the propagation direction of the incident wave, the scattering is determined mainly by amplitude functions.
- The numerical results obtained for plane mechanical systems in a particular case are compared with known values. In short waves $(h/\lambda > 0{,}5)$ the results differ to 10-15%, and in long waves $(h/\lambda > 0{,}5)$ до 25%

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