

Sound Propagation in a Square Duct having an Inlet and Outlet Located on the Crossed Right Angle Face

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ABSTRACT

A previous study discussed a rectangular sound proofing ventilation unit having an inlet and an outlet located on the opposite face which results in a reduced outlet area. As a result, the ventilation function is decreased. To improve this problem, square one featuring an inlet and an outlet which are located on the crossed right angle face is proposed in this work. First, outlet sound pressure to the given inlet uniform velocity is obtained based on the wave equation and the boundary conditions. Next, the experimental results are shown to be in reasonable agreement with our theoretical predictions.

Keywords: Wave equation, sound propagation, resonance, higher-order mode

INTRODUCTION

Along with the remarkable economic development, in recent years in developing countries environmental problems including road noise have been getting worse in countries belonging to tropical regions. Yano and others [1] measured road traffic noise on the main road of Vietnam for 24 hours and reported that a noise level of more than 75 dB occurred almost all day on the road with the highest noise level. On the other hand, ventilation holes are widely used in countries in tropical regions, which are installed in the middle of the window and the roof.

However, the noise of the road passes through the ventilation hole and propagates to the living space. As a result, there is virtually no difference in the noise level between the room and the outside, and people living in an environment are increasingly uncomfortable with noise increasing year by year even though there are considerable familiarity and patience. In recent houses, a glass plate is indicated on the outside of the door ventilation hole to provide sound proofing measures. With this measures indoors only attenuation levels of only 10 dB - 15 dB can be obtained, but people are considerably reduced discomfort to noise. However, most residential houses in Southeast Asia do not have air-conditioning equipment,

therefore by closing the glass plate during the day, the ventilate the room indoors cannot be done. A previous study [2] discussed a rectangular sound proofing ventilation unit having an inlet and an outlet located on the opposite face which results in a reduced outlet area. As a result, the ventilation function is decreased. To improve this problem, square one featuring an inlet and an outlet which are located on the crossed right angle confront is proposed in this work. There are for two sorts of waves inside the chamber, namely the standing wave that propagates in the axial direction and the traverse wave that propagates in the radial direction. The traverse wave occurs in the high frequency range. Regarding the circular cylindrical chamber, the noise will have a tendency to increase in this frequency range, far from decreasing depending on the resonance of the higher modes.

In this paper, at first the outlet sound pressure to the given inlet uniform velocity is obtained based on the wave equation and the boundary conditions. Next, the experimental results are shown to be in reasonable agreement with our theoretical predictions.

METHOD OF ANALYSIS

Insertion Loss

Insertion loss IL defined by [3]

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$$IL = 10 \log \frac{W_r}{W_0} = 20 \log \left| \frac{U_1}{U_2} \right| \quad (1)$$

Here, W_r and W_0 are the radiated power at one point in space with or without the acoustic element inserted between that point and the source. The ratio of U_1/U_2 is equal to the D parameter of the four-pole parameters, as far as constant velocity source be concerned.

When three acoustic elements connected in series, as the sectional area of element 1 and 3 are sufficiently small to compare with those of element 2 of the D parameter of the whole system can be described by the following approximated equation

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} A_w & B_w \\ C_w & D_w \end{pmatrix} \begin{pmatrix} A_3 & B_3 \\ C_3 & D_3 \end{pmatrix}$$

$$D = (\cos kl_1)(C_w)(jZ_3 \sin kl_3) \quad (2)$$

where C_w denoted the C parameter of element 2. In order to obtain a reliable IL effect, D parameter must be high enough. In other words, the design of element-2 to have a high enough parameter C_w is demanded.

C_w of square SVU

Model of the rectangular soundproofing ventilation unit which has a dimension of $a \times a \times d$ is shown in Figure 1. Dimension of an input and output are $(a_{i2} - a_{i1}) \times (b_{i2} - b_{i1})$ and $(a_{o2} - a_{o1}) \times (d_{o2} - d_{o1})$ they located on the face which has a section area of $S_{ab} = a \times a$ and $S_{ad} = a \times d$, respectively.

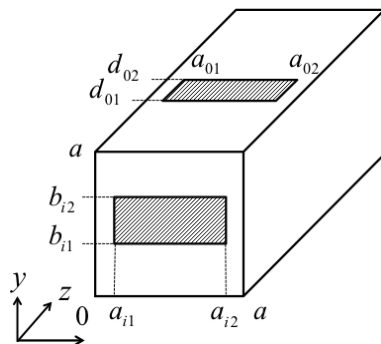


Figure 1. Square sound proofing ventilation unit

Wave equation in terms of velocity potential Φ is given by

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \Phi \quad (3)$$

where c is the sound velocity. Let $\Phi = \sqrt{2} \phi \exp(j\omega t)$ ($j^2 = -1, \omega = kc, k$: wave number) then the general solution of Eq.(3) can be given as

$$\begin{aligned} \Phi &= (A \exp(\mu Z) + B \exp(-\mu Z)) \\ &\times (C \sin ax + D \cos ax) \end{aligned}$$

$$\times (E \sin \sqrt{s^2 - \alpha^2} y + F \cos \sqrt{s^2 - \alpha^2} y) \quad (4)$$

where $\mu^2 = s^2 - k^2$, A, B, C, D, E and F are arbitrary constants determinable from the boundary conditions, other symbols are constants.

Let $-\partial\Phi/\partial x, -\partial\Phi/\partial y$ and $-\partial\Phi/\partial z$ are the velocity component in the x, y and z directions, respectively. Assuming the walls of the cavity to be perfectly rigid and the loss at the wall can be neglected then boundary conditions are :

$$[1] \text{ at } x=0 \quad -\frac{\partial\Phi}{\partial x} = 0 \quad (5)$$

$$[2] \text{ at } x=a \quad -\frac{\partial\Phi}{\partial x} = 0 \quad (6)$$

$$[3] \text{ at } y=0 \quad -\frac{\partial\Phi}{\partial y} = 0 \quad (7)$$

$$[4] \text{ at } y=a \quad -\frac{\partial\Phi}{\partial y} = V_0 F_0(x, z) \quad (8)$$

$$[5] \text{ at } z=0 \quad -\frac{\partial\Phi}{\partial z} = V_i F_i(x, z) \quad (9)$$

$$[6] \text{ at } z=d \quad -\frac{\partial\Phi}{\partial z} = 0 \quad (10)$$

where V_i is the driving velocity at the input, $F_i(x, z)$ is defined as

$$F_i(x, y) = \begin{cases} 1 & (a_{i1} \leq x \leq a_{i2}, b_{i1} \leq y \leq b_{i2}) \\ 0 & \text{other areas of } x \text{ and } y \end{cases} \quad (11)$$

Let $\Phi = \Phi_a + \Phi_b$, then $-\partial\Phi/\partial x = -\partial\Phi_a/\partial x - \partial\Phi_b/\partial x$, therefore we have the following boundary conditions

$$[1a] \text{ at } x=0 \quad -\frac{\partial\Phi}{\partial x} = 0 \quad (12)$$

$$[2a] \text{ at } x=a \quad -\frac{\partial\Phi}{\partial x} = 0 \quad (13)$$

$$[3a] \text{ at } y=0 \quad -\frac{\partial\Phi}{\partial y} = 0 \quad (14)$$

$$[4a] \text{ at } y=a \quad -\frac{\partial\Phi}{\partial y} = V_0 F_0(x, z) \quad (15)$$

$$[5a] \text{ at } z=0 \quad -\frac{\partial\Phi}{\partial z} = 0 \quad (16)$$

$$[1b] \text{ at } x=0 \quad -\frac{\partial\Phi}{\partial x} = 0 \quad (17)$$

$$[2b] \text{ at } x=a \quad -\frac{\partial\Phi}{\partial x} = 0 \quad (18)$$

$$[3b] \text{ at } y=0 \quad -\frac{\partial\Phi}{\partial y} = 0 \quad (19)$$

$$[4b] \text{ at } y=a \quad -\frac{\partial\Phi}{\partial y} = 0 \quad (20)$$

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$$[5b] \text{ at } z=0 - \frac{\partial \phi}{\partial z} = V_i F_i(x, z) \quad (21)$$

Based on the above boundary conditions ϕ_a becomes

$$\phi_a = 4 \frac{V_i}{S_{ab}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\cosh \mu_{m,n}(z-d)}{\mu_{m,n} \sinh \mu_{m,n} d} I_{m,n} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{a}\right) \quad (22)$$

where

$$I_{m,n} = \int_{a_{i1}}^{a_{i2}} \cos\left(\frac{m\pi x}{a}\right) dx \times \int_{a_{i1}}^{a_{i2}} \cos\left(\frac{n\pi y}{a}\right) dy \quad (23)$$

Similarly, ϕ_b becomes

$$\phi_b = 4 \frac{V_0}{S_{ad}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \cosh \frac{n\pi}{d}(z-d) \times O_{m,n} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{a}\right) \quad (24)$$

where

$$O_{m,n} = \int_{a_{i1}}^{a_{i2}} \cos\left(\frac{m\pi x}{a}\right) dx \times \int_{a_{i1}}^{a_{i2}} \cos \frac{n\pi}{d}(z-d) dy \quad (25)$$

The average sound pressure on the output is found as

$$\begin{aligned} \bar{P}_0 &= \frac{1}{S_0} \int_{a_{02}}^{a_{01}} \int_{d_{01}}^{d_{02}} P(x, b, z) dx dz \\ &= jk \frac{\rho c}{S_0} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[4 \frac{V_i}{S_{ab}} \frac{I_{m,n} \cos(n\pi)}{\mu_{m,n} \sinh \mu_{m,n} d} \right. \\ &\quad \times \int_{a_{02}}^{a_{01}} \int_{d_{01}}^{d_{02}} \cos \mu_{m,n}(z-d) \cos\left(\frac{m\pi x}{a}\right) dx dz \\ &\quad \left. + 4 \frac{V_0}{S_{ad}} \frac{O_{m,n}}{\beta_{m,n} \tan \beta_{m,n} d} \right] \\ &\quad \times \int_{a_{02}}^{a_{01}} \int_{d_{01}}^{d_{02}} \cos \frac{n\pi}{d}(z-d) \cos\left(\frac{m\pi x}{a}\right) dx dz \quad (26) \end{aligned}$$

Where $U_i = V_i S_i$ is the volume velocity supplied from the input. $Z_0 = \rho c / S_0$ is the characteristic impedance of the output.

Expanding Eq.(26) with $m=0$ and $n=0$ the above equation becomes

$$\bar{P}_0 = j4Z_0 \left[\frac{1}{\sin kd} \left(-O_{0,0} U_i + \left(\frac{S_0}{S_{ad}} \cos kd \right) U_0 \right) \right]$$

$$\begin{aligned} &+ \frac{1}{k} \sum_{*}^{\infty} \sum_{*}^{\infty} \left(\frac{I_{m,n} O_{m,n} \cos(n\pi)}{S_{ad} S_0 \mu_{m,n} \sinh \mu_{m,n} d} U_i \right. \\ &\quad \left. + \frac{U_0}{S_{ad} S_0} \frac{O_{m,n}}{\beta_{m,n} \tan \beta_{m,n} d} \right) \quad (27) \end{aligned}$$

The C_w of SVU could be found from Eq. (27) as

$$C_w = U_i / \bar{P}_0 \quad (28)$$

RESULTS AND DISCUSSION

The average of outlet sound pressure is derived from Eq. (27) in which the first term in the bracket represents the plane wave and the second represents the higher order mode wave. First, with respect to the plane wave, the first term of Eq. (27), the sound pressure level is great at the frequencies where the denominator $\sin(kd)$ become zero.

Namely

$$kd = \eta\pi \therefore f = \eta \frac{c}{2d} (\eta = 1, 2, \dots) \quad (29)$$

Similarly, with respect to higher order mode waves, the second term of Eq. (27), the sound pressure level becomes great at the frequencies given as follows:

$$\sinh \mu_{m,n} d = 0$$

$$\begin{aligned} \therefore f_{m,n} &= \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2 + \left(\frac{\eta\pi}{d}\right)^2} \\ &= \frac{c}{2a} \sqrt{m^2 + n^2 + \left(\frac{\eta a}{d}\right)^2} \end{aligned}$$

$$(\eta = 0, 1, 2, \dots) \quad (30)$$

The spectrum of a plane wave and higher order mode are showed in Figure 2.

Note that, in the case of rectangular SVU, the resonance frequencies are

$$f_{m,n} = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\eta\pi}{d}\right)^2}$$

$$(\eta = 0, 1, 2, \dots) \quad (31)$$

From Eq. (30) and Eq. (31), it is clear that the number of resonance frequencies generated inside the square SVU is less than those of rectangular SVU.

Verifying the theoretical results

As described above, the waves in the SVU is

composed of a plane wave and a higher order mode wave. In order to maximize the soundproofing ability of the SVU, it is necessary to reduce the sound pressure level or to prevent those waves from even occurring.

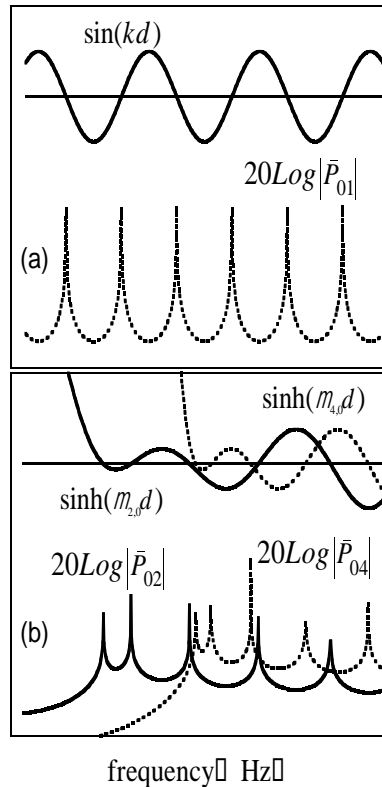


Figure 2. Resonance frequencies appear in the SVU. Resonances will occur when the denominator of Eq.(27) become zero. (a) plane wave component. (b) higher order mode wave component.

When we locate the center of the outlet at $z = d/2$ Eq. (27) becomes

$$\begin{aligned} \bar{P}_0 = j4Z_0 \left[\frac{1}{\sin\left(\frac{kd}{2}\right)} \left(-O_{0,0}U_i \right) \right. \\ \left. + \left(\frac{S_0 \cos kd}{S_{ad} \cos\left(\frac{kd}{2}\right)} \right) U_0 \right] \\ + \frac{1}{k} \sum_{*}^{\infty} \sum_{*}^{\infty} \left(\frac{I_{m,n} O_{m,n} \cos(n\pi)}{S_{ad} S_0 \mu_{m,n} \sinh\left(\frac{\mu_{m,n}d}{2}\right)} U_i \right. \\ \left. + \frac{U_0}{S_{ad} S_0} \frac{O_{m,n}}{\beta_{m,n} \tan \beta_{m,n} d} \right) \end{aligned} \quad (32)$$

With respect to the plane wave, the first term of Eq. (32), the sound pressure level is great at the frequencies where the denominator $\sin(kd/2)$

become zero.

Namely

$$\frac{kd}{2} = \eta\pi \therefore f = \eta \frac{c}{d} (\eta = 1, 2, \dots) \quad (33)$$

Similarly, with respect to higher order mode waves, the second term of Eq. (32), the sound pressure level becomes great at the frequencies given as follows:

$$\begin{aligned} \sinh(\mu_{m,n}d/2) = 0 \\ \therefore f_{m,n} = \frac{c}{\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2 + \left(\frac{\eta\pi}{d}\right)^2} \\ = \frac{c}{a} \sqrt{m^2 + n^2 + \left(\frac{\eta a}{d}\right)^2} \end{aligned} \quad (\eta = 0, 1, 2, \dots) \quad (34)$$

From the above equations we can understand that the $\sin(kd)$ and $\sinh(\mu_{m,n}d)$ of \bar{P}_0 in Eq. (31) becomes $\sin(kd/2)$ and $\sinh(\mu_{m,n}d/2)$, respectively. This results in a halving of the number of resonance frequencies generated inside the SVU.

Figure 3 shows the experiment results to prove this fact. The dotted line is the measured result when the microphone was located on the position of $z=d$. The solid line is the measured result when the microphone was located on the position of $z=d/2$. Note the measurement method was based on reference [4].

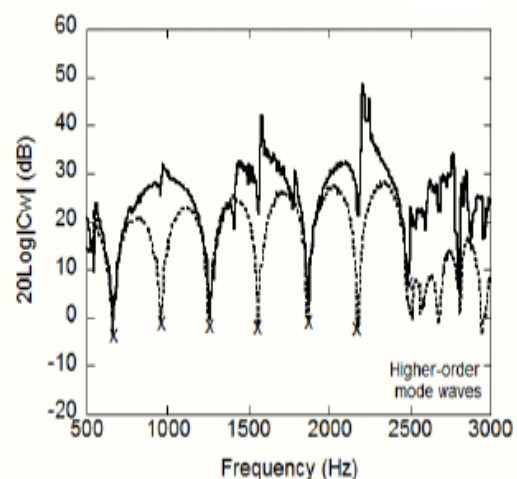


Figure 3. Measured result ($a=0.14m$, $d=0.7m$). Dotted line and solid line are the output located at d and $d/2$, respectively.

CONCLUSION

The presented here is the sound propagation in the square SVU featuring an inlet and outlet is

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located on crossed right angle face. The outlet position was improved by placing it on the larger face of the SVU to increase the ventilation effect. In addition, the outlet is located in the center of the SVU to avoid the resonance of plane wave sound pressure.

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Citation: Sohei Nishimura, Yuya Nishimura, (2018). "Sound Propagation in a Square Duct having an Inlet and Outlet Located on the Crossed Right Angle Face", *International Journal of Emerging Engineering Research and Technology*, 6(8), pp. 9-13.

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