International Journal of Emerging Engineering Research and Technology
Volume 2, Issue 6, September 2014, PP 158-162
ISSN 2349-4395 (Print) & ISSN 2349-4409 (Online)

The Polygon Model of Rolling Friction

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Abstract: *This model calculates the coefficient of rolling friction of a wheel by assuming the energy involved in a rolling polygon inscribed in a circle represented by the deformation the wheel on a level horizontal un deformed surface. Here it is assumed that elastic potential energy stored in wheel due to deformation at the contact is not conserved in motion, since the displacement is always normal to elastic force.*

Keywords: *Rolling friction, coefficient of rolling friction, polygon model of rolling friction*

1. Introduction

Rolling friction is one of the basic phenomena man encounters in his everyday life since ancient times when the wheel was invented. The phenomenon of rolling friction has been interesting to scientists for a long time. Scientific publications on this subject range back to 1785 when Vince described systematic experiments to determine the nature of friction laws [1], and important scientists dealt with this problem, among them Reynolds [2]. The rolling friction is of great importance in engineering and science. For its major importance the phenomenon has been studied intensively by engineers and physicists [3-8], however, surprisingly little is known about its basic mechanisms. To my knowledge there is still no "first-principle" expression for the rolling friction coefficient available which relates this coefficient only to the material constants of the rolling body and does not contain empirical parameters. It has been shown that surface effects like adhesion [6], electrostatic interaction [7], and other surface properties [8] might influence the value of the rolling friction coefficient. Theoretically this problem was studied in ref. [9] where the authors propose a model of a surface with asperities to mimic friction (see also [10]). In other studies [11], [12] it was argued that for viscoelastic materials the rolling friction is due very little to surface interactions: the major part is due to deformation losses within the bulk of the material" [11]. Based on this concept the rolling friction coefficient was calculated in [11] where the deformation in the bulk was assumed to be completely plastic; then an empirical coefficient was introduced to account for the retarded recover of the material. Dupuit's inverse square root relationship [13] assumes that the coefficient of rolling friction is directly proportional to squirt of load and inversely proportional to the squire root radius of the wheel. The approach of present model focuses up on energy dissipated on completely damped rolling motion.

2. The Polygon Model

The coefficient of rolling friction for a slow moving tyre on hard non deformable smooth surface can be theoretically calculated by the modelling method adopted here. When the speed of rotation is very low we can consider complete damping and the coefficient of rolling friction depends only on the extent of deformation for a given radius. The extent of deformation is inversely related to tyre pressure or modulus of elasticity of the material (in case of sold wheel) also on width of the tyre and directly related to load.

Consider a tyre deformed by a depth 'h' as shown in fig 1

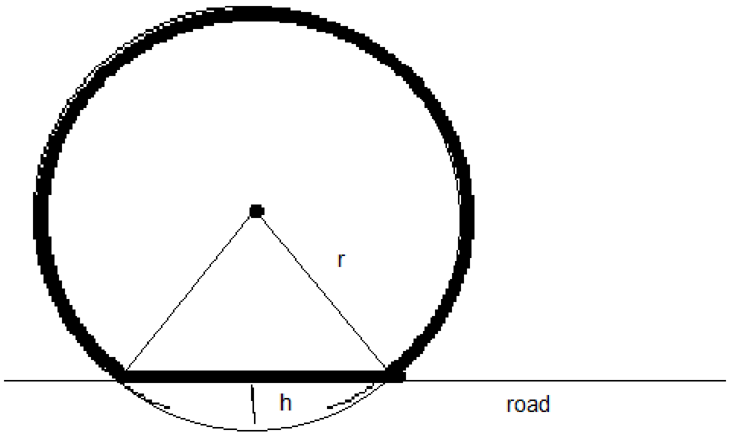

Fig1

Here the frictional work done in a cycle is imagined to be equal to sum of work done against gravity by rolling of regular polygon inscribed in a given circle described by the depth of deformation ‘h’. The number of sides of the inscribed polygon ‘n’ depends on the value of ‘h’ for a wheel of given radius ‘r’. When depth of deformation ‘h’ becomes zero polygon reduces to a circle.

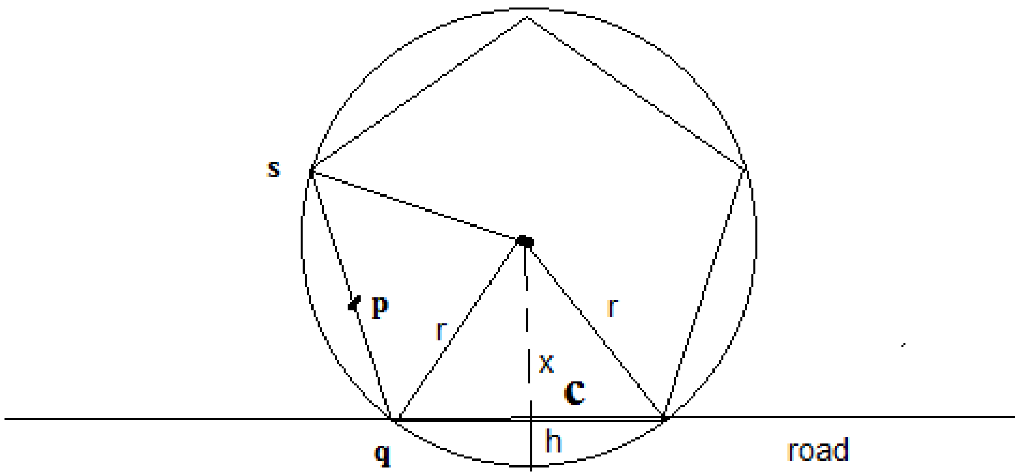

Fig2

Now consider a regular pentagon, the sum of work done against gravity by the a regular pentagon in a complete cycle on smooth level surface is equal to ‘5mgh’, where ‘m’ is the mass of the system and ‘g’ is the acceleration due to gravity. Here work done by gravity “mgh” is stored in the form elastic potential energy and is not conserved in motion, since the displacement is always normal to elastic force. The elastic energy is assumed to be converted to frictional heat (produced due to vibrational damping) and another part is converted to energy required plastic deformation, wear and tear.

In general, for an ‘n’ sided polygon, work done on rolling a cycle against gravity is equal to n.mgh, where the deformation depth ‘h’ becomes smaller when the number of sides of polygon ‘n’ becomes larger.

The concept of rolling polygon model can be justified from the fact that the distance moved by the wheel in one complete turn of the axle will be equal to the sum of the side lengths of the polygon (n x c). The distance advanced in one complete turn of the axle is equal to ‘2πr’ only when the wheel has no deformation.

The concept of rolling polygon model can also be justified from the fact that the number of elastic hysteresis curves per cycles is equal to number of sides of polygon. From the fig 2 the elastic loading at point ‘p’ starts when the point q just touches the ground, the loading becomes a maximum when the point p touches the ground. Unloading at point ‘p’ takes place on further rolling and unloading get completed when the point ‘s’ just leaves the ground. The elastic hysteresis can be depicted as follows

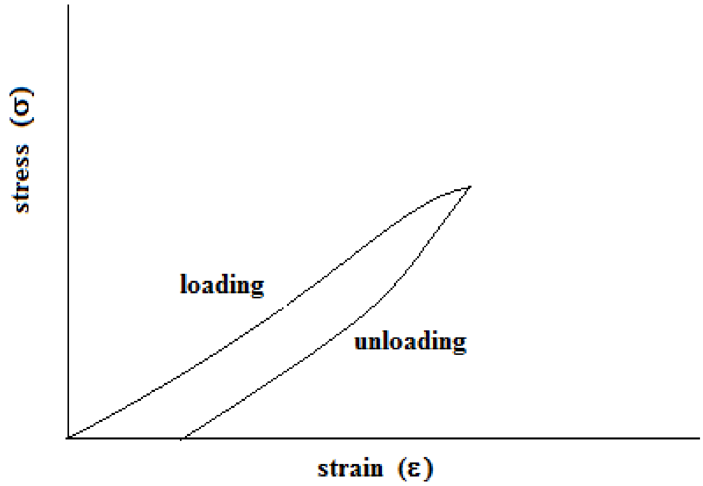

Fig3.

In the case of rubber tyre the hysteresis area is much larger than steel therefore good damping is obtained.

From the fig. 2

$$r = x + h \tag{1}$$

On applying law of cosines

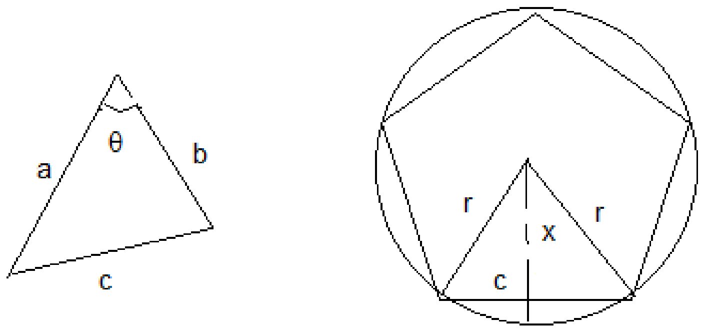

Fig4.

According to law of cosines for a triangle of sides a,b,c

$c^2 = a^2 + b^2 - 2abcos\theta$ Where 'θ' is the angle opposite to side c

On applying law of cosines to regular polygon of 'n' side of edge length 'c'

$c^2 = r^2 + r^2 - 2r^2\cos(\frac{2\pi}{n})$ or

$$c = r\sqrt{2 - 2\cos(\frac{2\pi}{n})}$$

$$c = r\sqrt{2(1 - \cos(\frac{2\pi}{n}))}$$

$$c = r\sqrt{2 \times 2\sin^2(\frac{\pi}{n})}$$

$$c = 2rsin(\frac{\pi}{n}) \tag{2}$$

From eq. (1)

$$h = r - x$$

$$h = r - \sqrt{r^2 - \left[\frac{c}{2}\right]^2}$$

Substituting eq. (2)

$$h = r - \sqrt{r^2 - \frac{4r^2 \sin^2(\frac{\pi}{n})}{4}}$$

Or $h = r - r\sqrt{1 - sin^2}(\frac{\pi}{n})$

$$h = r(1 - cos(\frac{\pi}{n}))$$

Or $h = 2rsin^2(\frac{\pi}{2n})$ (3)

Since frictional work done 'W' in a cycle is equal to sum of work done against gravity by rolling polygon

$$W = nmgh = nmg2rsin^2(\frac{\pi}{2n})$$

Let be F is the frictional force then

$$F = \frac{W}{distance} = \frac{W}{nc}$$

The coefficient of rolling friction μ is given by

$$\mu = \frac{F}{mg} = \frac{W}{ncmg} = \frac{2nmgrsin^2(\frac{\pi}{2n})}{nmg2rsin(\frac{\pi}{n})}$$

$$\mu = \frac{1}{2}tan\left(\frac{\pi}{2n}\right) \quad (4)$$

or $\mu = \frac{1}{2}tan\left(\frac{\theta}{4}\right)$

Table1

n	1	2	3	4	5	6	10	100	1000	∞
μ	∞	0.5	0.2886	0.2071	0.1624	0.1339	0.07919	0.007855	0.000785	0

The dragging force (F) involved in rolling friction is obtained by multiplying coefficient of rolling friction (μ) with the load (mg).

$$F = \mu.mg$$

3. RESULTS AND DISCUSSIONS

From the equation (4) it is seen that, as the number of sides of polygon increases the coefficient of rolling friction decreases. As the number of sides of the polygon diameter of the wheel and extent of deformation are related each other, the coefficient of friction is inversely related to diameter. Since the extent of deformation is related to load, the coefficient of rolling friction is related to load and is unlike coefficient of sliding friction. The extent of deformation decreases when there is increase in elastic modulus. The rolling friction of steel wheels of a train is much lower than rubber tyres of a truck. Similarly ceramic bail bearing offers much lesser rolling friction than steel. It can be also concluded that when 'n' is large, for every tenfold increase in number of sides there is one tenth decrease in the coefficient of rolling friction (table 1). This fact is also supported by an experimental data reported [14].

The elastic potential energy involved in rolling polygon is assumed to be not conserved; instead it gets converted to damped vibrational energy and converted to frictional heat.

Even if we assume a hard undeformable wheel and a deformable rolling surface, the above equation will holds well, based on energy considerations. If both surfaces are deformable then consider the total deformation depth to fit the polygon in the circular wheel of given radius.

When width of the wheel increases extend of deformation decreases but sliding movement increases when the wheel moves in a non-linear path. Hence the equation will not hold good for wide cylindrical wheels.

When the speed of revolution is high the energy involved in rolling is not damped completely into heat, some vibrational energy remains in the wheel which changes the deformation depth with time. If the rolling is not damped a component of translational kinetic energy gets converted to vibrational KE and amplitude of vibration rises. Here effects like plastic deformation of the material and mechanical welding of two surfaces may takes place due to impact and the present model will not work well.

When the surface is not smooth the coefficients of friction depends on load, speed, degree of roughness and extend of deformation. When extend of deformation is larger, phononic energy loss factor will be smaller than the hard less deformed tire, while moving on rough surface. Therefore right choice of air pressure inside a tire depends on road smoothness, if the road is smooth, less inflated tire is preferred. Therefore damping factor is also important like coefficient of friction in designing wheels.

The model can also be extended to toroid shaped wheels and spherical balls by considering the depth of deformation on contact points.

4. CONCLUSION

This model can be treated as “first-principle" expression for the rolling friction coefficient, which relates this coefficient only to the material constants of the rolling body or depth of deformation and does not contain empirical parameters.

REFERENCES

[1] Vince S., Philos. Trans. R. Soc. London, 75 (1785) 165.

[2] Reynolds O., Royal Society, Philos. Trans., 166 (1874) Pt. 1.

[3] Huang R. Q. and Wang S. T., in Proc. Intern. Symp. Landslides, Lausanne, 1988, edited by C. Bonnard, (1988), p. 187.

[4] Herrmann H. J., Mantica G. and Bessis D., Phys. Rev. Lett., 65 (1990) 3223.

[5] Heathcote H. L., Proc. Inst. Automot. Eng., 15 (1921) 569

[6] Barquins M., Maugis D., Blouet J. and Courtel R., Wear, 51 (1978) 375

[7] Deryaguin B. V. and Smilga V. P., Prog. Surf. Sci., 45 (1994) 108

[8] Chaplin R. L. and Chilson P. B., Wear, 107 (1986) 213; Song Q., Am. J. Phys., 56 (1988)1145

[9] Fuller K. N. G. and Roberts A. D., J. Phys. D, 14 (1981) 221.

[10] P¨oschel T. and Herrmann H. J., Physica A, 198 (1993) 441.

[11] Greenwood J. A., Minshall H. and Tabor D., Proc. R. Soc. London, Ser. A, 259 (1961) 480.

[12] Tabor D., Proc. R. Soc. London, Ser. A, 229 (1955) 198

[13] K.L. Johnson, Contact Mechanics, p. 306-311, Cambridge Univ. Press (1985)

[14] www.essential-physics.com/e-BookData/BookInd-98.html