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Design of Fractional Order Differentiator & Integrator Circuit Using RC Cross Ladder Network

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Abstract: *In this paper the concept of 'FRACTIONAL ORDER' element is reported. RC ladder network itself behaves as a fractional order element which is developed and the same ladder network is been used in Integer order differentiator and integrator circuit to make it 'Fractional order differentiator' (FOD) and 'Fractional order integrator' (FOI) circuit. The performance of FOD and FOI using RC ladder network is studied in both frequency & time domain. Same response is then compared with the performance of Integer order differentiator and integrator circuit. The simulation of FOD and FOI is done using NI Multisim 12.0 software.*

Keywords: *Fractional order element; ladder network; differentiator; integrator.*

1. INTRODUCTION

Research for Fractional order is been carried out for more than 300 years. Numbers of real objects and dynamic systems are been modeled accurately or precisely with fractional calculus [1], [2], [3], [4]. Hence, applications of fractional calculus are increasing rapidly and fractional calculus has attracted more and more attention of researchers. Such as diffusion of heat through semi-infinite solid, where heat flow equals to the half-derivative of the temperature [5] and most recent biological systems[6], [7]. Earlier only Integer order models were used because solution methods in the form of Fractional differential equations were not developed. But at present situation lots of solution methods are available in the form of fractional differential & fractional integral. So that fractional calculus can be easily calculated & used in wide range of applications such as Modern control theory, electrical circuits etc.[8].

Expansion of calculus to Fractional order had strong & firm weitage in foundation theory. Leibniz proved same thing with L'Hospitality rule over 300 years ago (1695). Some systematic study also been made by Liouville (1832), Holmgren (1864) & Riemann (1953) in the middle & beginning of 19th century. Eurel & Lagrange also made their contribution even earlier [9], [10]. Some prominent works were also carried out around 60's [11]. However, FOC systems were not used due to the absence of solutions in the form of Fractional differential & integral equations.

In last few decades, researchers have found that various materials & dynamic processes can be modeled very precisely using Fractional order differential equations [8], [12], [10]. The fact of 'Fractional order models need fractional controller for effective control' has attracted researcher's attention towards various applications of Fractional order systems [2], [14], [15], [16]. Reasearchers of Fractional order control are mainly centered in Europian universities. The CRONE (non-integer robust control) team in France is leaded by Alain Oustaloup & Patrick Lanusse from Bordeaux University, France. Their experiments include application of 'Fractional order control' in car suspension control & hydraulic actuator [17]. Denis Matignon, a researcher from ENST, signal department & CRNS added his theoretical concepts related to FOC such as stability, contrallability & observability [7]. In this paper a very simple, low cost and easy to use fractional order element is developed and the performance of fractional order differentiator and integrator has been observed.

The paper is organized as follows: section 2 provides background of fractional order element, section 3 gives idea about performance of designed fractional order element. Section 4 shows performance of fractional and integer order differentiator circuit. Section 5 provides idea about fractional and integer order integrator circuit Finally, in section 6 discussions and conclusions have been looked into.

2. BACKGROUND

The impedance of any passive circuit element can be expressed by,

$$= Z(s) = Qs^{-\alpha} \quad (1)$$

Where Z is the impedance, Q is a constant, α is a real number and s is the Laplace operator. So,

$$|Z| = Q\omega^{-\alpha} \quad \text{and} \quad < z = -\frac{\alpha\pi}{2} \quad (2)$$

When $\alpha = 1$, $Z = Qs^{-1}$ and Z represents a capacitor. Similarly for $\alpha = 0$ and -1, Z represents a resistance and inductance respectively [20]. In the case when α is a fraction, then Z is a fractional order element (FOE).

From the Equation (2) it is found that, the magnitude depends on both frequency, ω and α. But it is evident that phase angle is independent of the frequency. It can also be said that an ideal capacitor has a phase angle of -90^0, an inductance has 90^0 and resistance has 0^0 phase angle. An FOE can have any phase angle within -90^0 to 0^0 or 0^0 to $+90^0$.

3. PERFORMANCE STUDY OF FRACTIONAL ORDER ELEMENT

RC ladder network is been developed shown in Fig1. From experimental practices, -45˚ phase angle is been provided by RC ladder network. Infinite stages of such network can be configured but for convenience, only five stages have been shown. This phase angle remains constant over particular range of frequencies hence also known as 'Constant phase element'. Other names for FOE are Fractance, Fractor etc. [18], [19], [20]. For phase angle to be -45˚, values of resistors & capacitors considered such as R = 22KΩ, C = 0.1μF. The infinite cross ladder network gives phase impedance,

$$Z(s) = \left(\frac{R}{sC}\right)^{0.5} \quad (3)$$

Comparing equation (1) with (3), it can be said that α = 0.5 [21]. As order of transfer function is 0.5 which is fractional; circuit also known as 'Fractional Order Element'. It is possible to obtain different values of 'α' but for those values of R & C are needed to be changed.

The readings of impedances (Z) and phase angles (θ) of the RC ladder network are recorded by varying the frequency from 100 Hz to 1.5 kHz. The instrument used to measure the impedance characteristics of the ladder circuit is an LCR meter (Agilent Impedance analyzer 4294A) in Z, θ mode, excited with a sinusoidal signal of 2V peak to peak.

Fig(2) and (3) show frequency response of RC cross ladder network. It is clear from Fig(3) that, constant phase angle is achieved between 0.4 to 0.8 kHz frequency ranges & hence 'constant phase behavior' is achieved as suggested by its name. The change of magnitude and phase angle in this zone of frequency is plotted in Fig(2) and (3). The constant phase obtained experimentally from cross RC ladder network is -45˚ and Q value is 4.51×10^5 which is obtained from Equation (1).

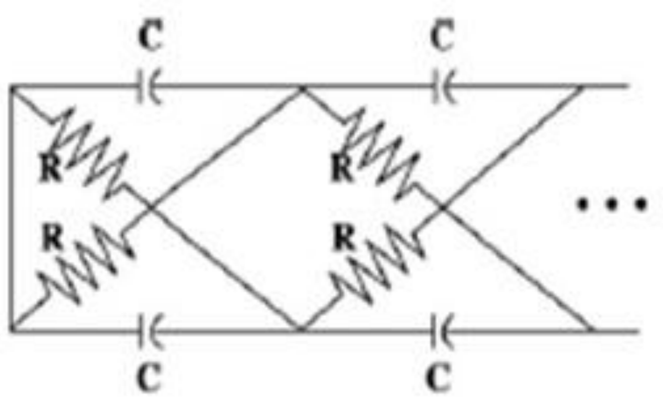

Fig1. *RC Cross l adder network*

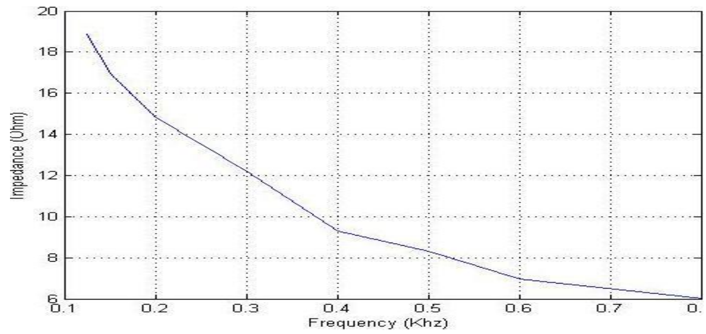

Fig2. *Magnitude response of RC ladder network*

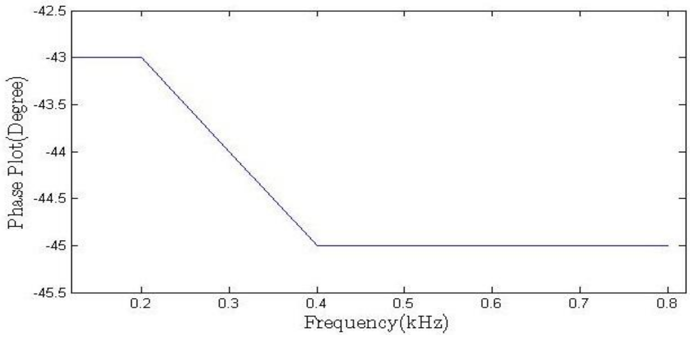

Fig3. *Phase plot RC Cross ladder network.*

4. PERFORMANCE STUDY OF FRACTIONAL AND INTEGER ORDER DIFFERENTIATOR CIRCUIT

Fractional order differentiator is been constructed using RC ladder network (FOE). Ideal capacitor is been replaced with RC ladder network & this makes fractional order differentiator (FOD). Its performance in both frequency & time domain is been studied in this section. Fig4 shows circuit diagram of fractional order differentiator. In the Fig4 FOE refers to constant phase element which is nothing but the five stage RC ladder network.

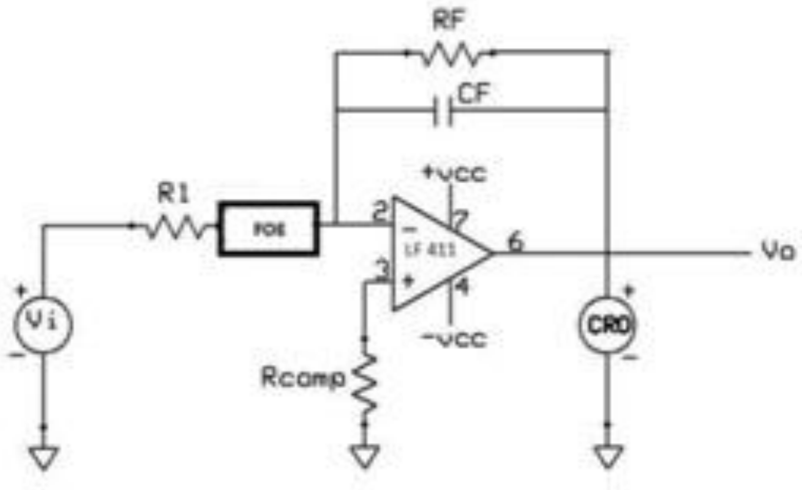

Fig4. *Fractional order differentiator*

4.1. Frequency and Time Response of Fractional Order Differentiator (FOD) Circuit with Sinusoidal Input

Frequency response of FOD with RC cross ladder network i.e. gains and phase angle curve are shown in Fig5 and Fig6.

In FOD circuit Input signal is been given by function generator with 2 volt peak to peak and input frequency range is given within the range of 0.125 Hz to 1.5 kHz.

Fig5 shows magnitude plot whereas Fig6 shows Phase plot of FOD circuit.

From magnitude response it can be said that, if we ignored the beginning portion of plot then rest of the portion of plot is decaying in nature as that of magnitude plot of FOE. From phase response it can be said that, phase angle of -45° is constant within the range of 0.3 kHz 1.4 kHz.

Fig7 shows the time response of experimentally obtained input and output waveforms for the FOD circuit. The input and output waveforms obtained using oscilloscope model no.3806 APLAB.

Fig8 shows output waveform obtained by simulation. NI Multisim 12.0 software is been used here for simulation.

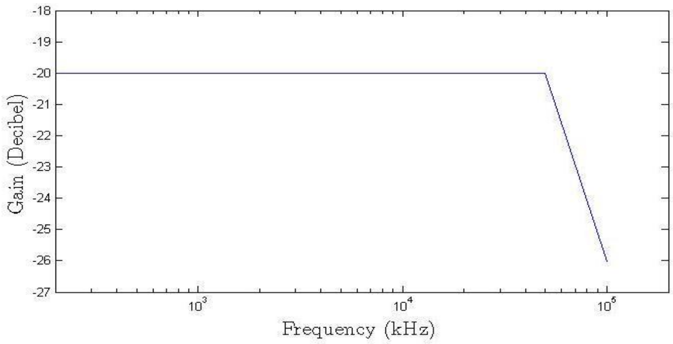

Fig5. *Magnitude response of FOD circuit with RC cross ladder network.*

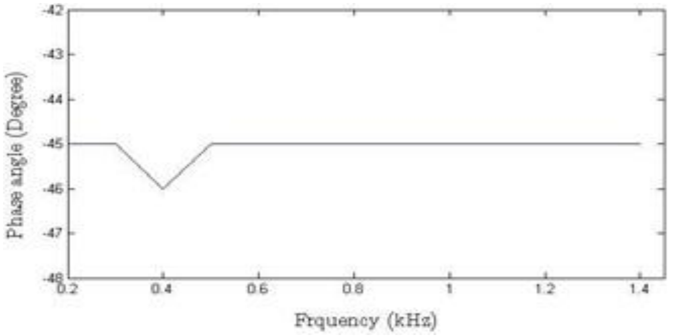

Fig6. *Phase response of FOD circuit with RC cross ladder network*

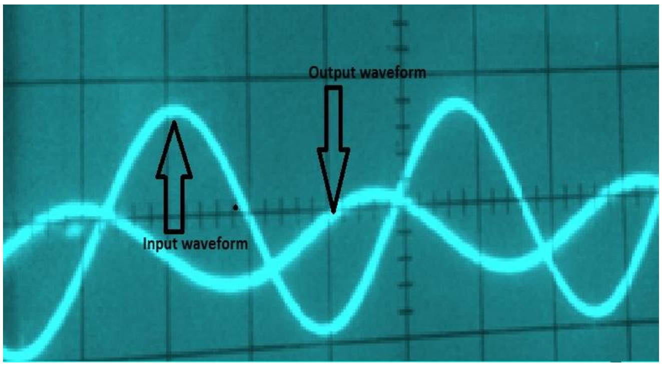

Fig7. *Time response for FOD*

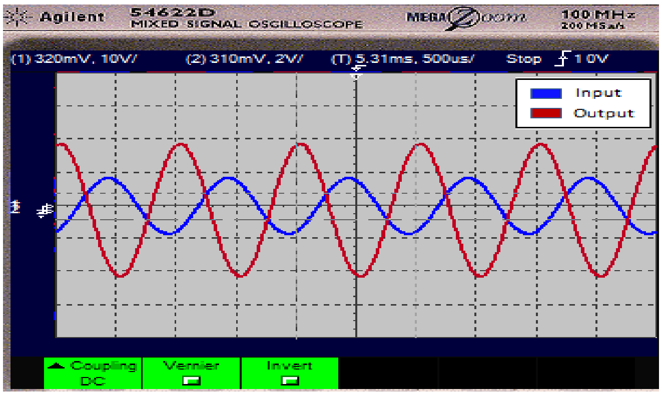

Fig8. *Simulated Output for FOD*

4.2. Frequency and Time Response of Integer Order Differentiator (IOD) Circuit with Square Input

Frequency and time response of the integer order differentiator circuit are observed with the same CRO as that of FOD mentioned above. Fig9 shows the magnitude plot of IOD whereas Fig10 shows phase plot of IOD. Magnitude plot of IOD circuit is rising in nature in the beginning which satisfies the characteristic of IOD.

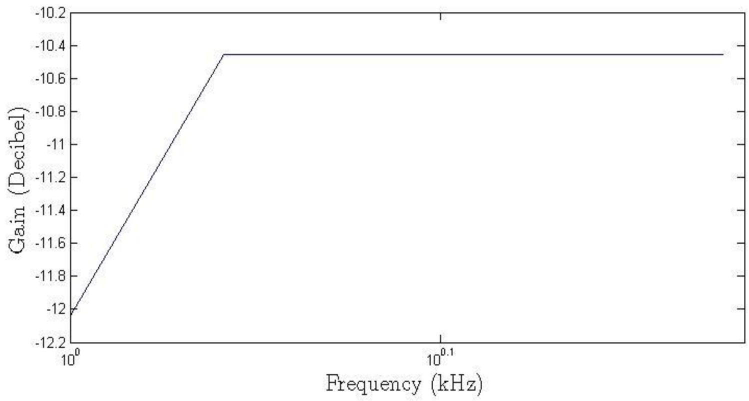

Fig9. *Magnitude response of IOD circuit*

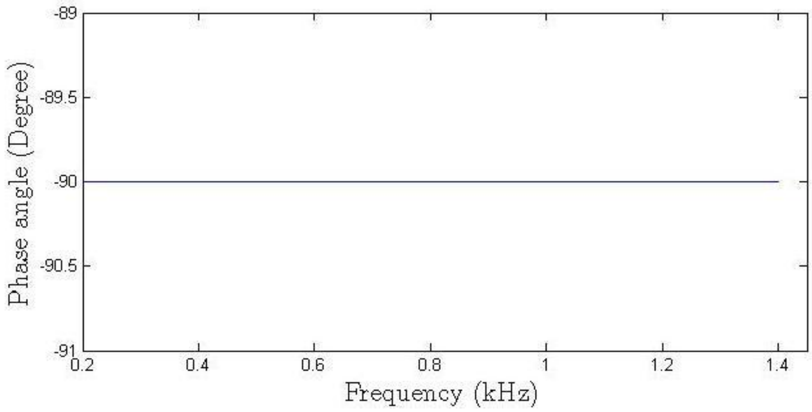

Fig10. *Phase response of IOD circuit*

It can be seen from Fig10 that constant phase angle of -90° is obtained over the full range of frequency which is nothing but again the characteristics of IOD.

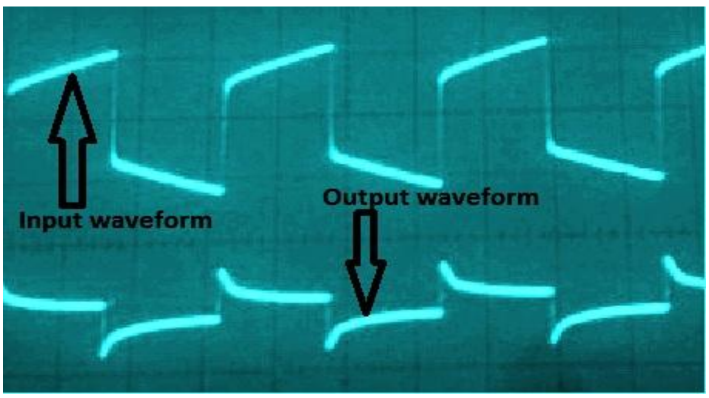

Fig11. *Time response for IOD*

Fig11 shows experimentally obtained input & output waveforms for IOD. Same oscilloscope is being used here as that of FOD.

5. Performance Study of Fractional and Integer Order Integrator Circuit

Fractional order integrator is been constructed using RC ladder network (FOE). Ideal capacitor is been replaced with RC ladder network & this makes fractional order integrator (FOI) circuit as shown in Fig12. Its performance in both frequency & time domain is been studied in this section.

5.1. Frequency and Time Response of Fractional Order Integrator (FOI) Circuit with Sinusoidal Input

Frequency response of FOI with RC cross ladder network i.e. gains and phase angle curve are shown in Fig13 and 14.

In FOI circuit Input signal is been given by function generator with 2 volt peak to peak and input frequency range is given within the range of 20 to 100 kHz.

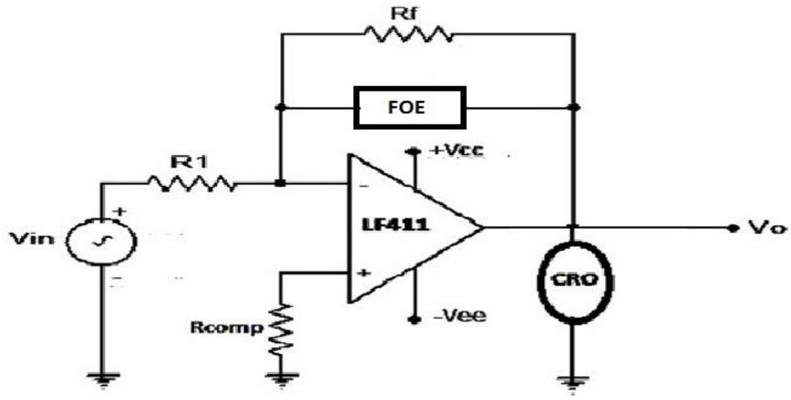

Fig12. *Fractional order integrator*

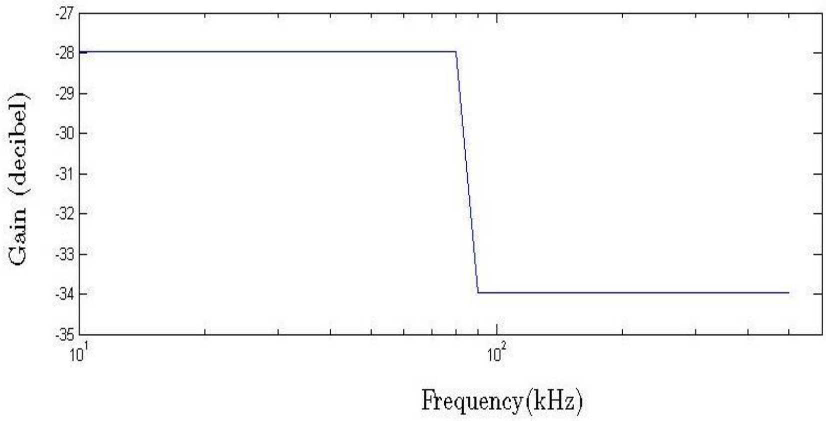

Fig13. *Magnitude response of FOI circuit with RC cross ladder network*

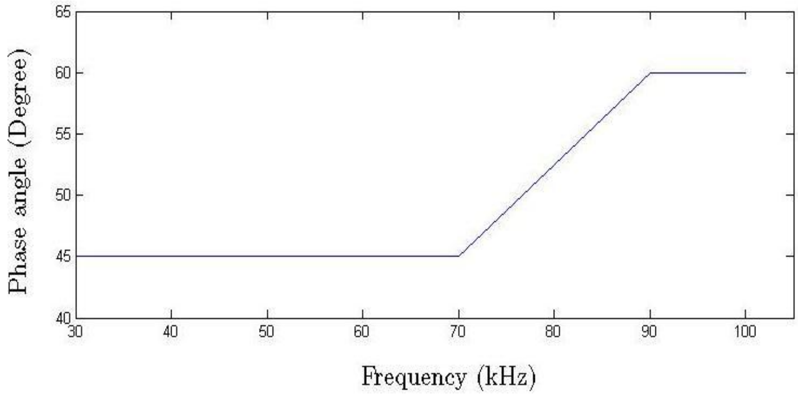

Fig14. *Phase response of FOI circuit network*

Fig13 shows magnitude response of FOI circuit. Magnitude response is constant within the range of 10 to 70 kHz then it is decaying from 70 to 80 Hz and again becomes constant from 80 to 500 kHz. Fig14 shows phase plot of FOI circuit. As we can see in this diagram, constant phase angle of -45° is obtained within the range of 30 to 70 kHz. So it can be said that phase response of RC ladder network matches with the phase response of FOI circuit but the range of frequency is different.

Fig15 shows the time response of experimentally obtained input and output waveforms for the FOI circuit. Fig16 shows simulated waveform of FOI obtained by NI Multisim 12.0 software.

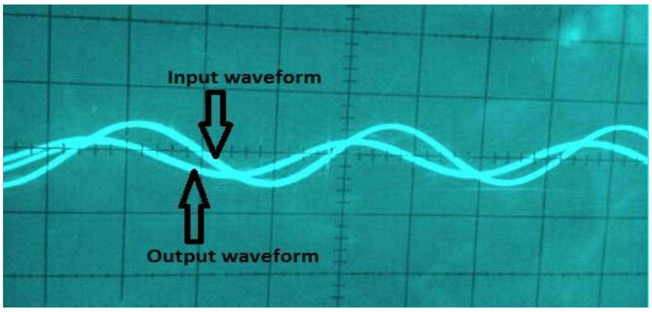

Fig15. *Time response of FOI*

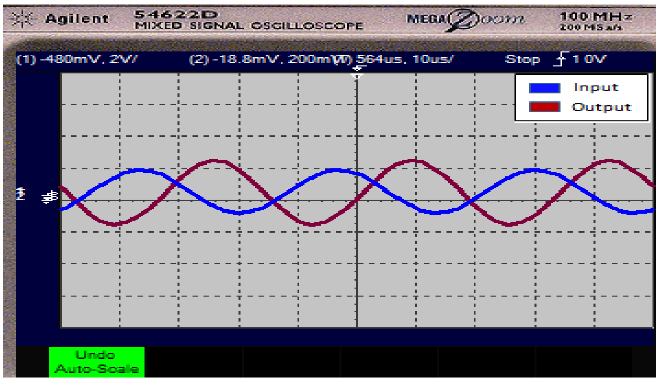

Fig16. *Simulated output for FOI*

5.2. Frequency and Time Response of Integer Order Integrator Circuit with Square Input:

Frequency and time response of the integer order integrator (IOI) circuit are observed with the same CRO as that of FOD mentioned above. Fig17 shows the magnitude plot of IOD whereas Fig18 shows phase plot of IOI Magnitude plot shows magnitude of IOI remains constant within the range of $10^{0.3}$ to $10^{0.75}$ kHz and again within $10^{0.75}$ to $10^{0.95}$ kHz but it is decaying between $10^{0.7}$ to $10^{0.8}$ kHz.

It can be seen from Fig18 that constant phase angle of -90° is obtained over the full range of frequency which is nothing but the characteristics of IOI.

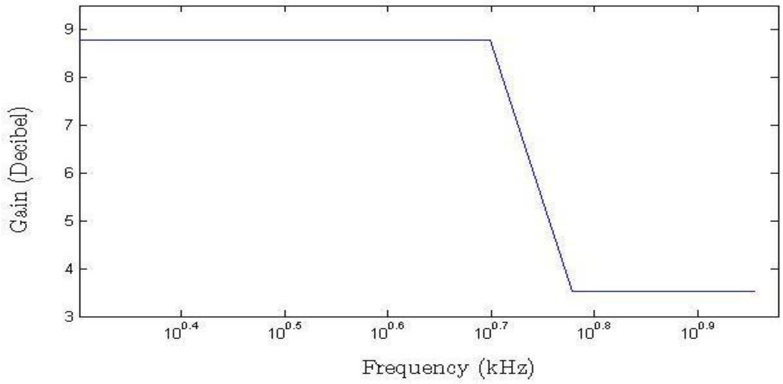

Fig17. *Magnitude response IOI circuit*

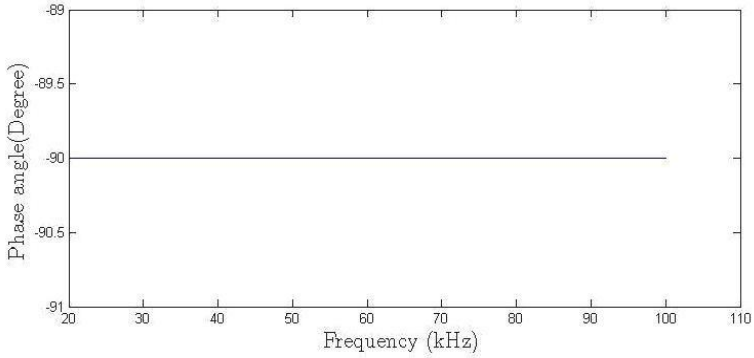

Fig18. *Phase response of IOI*

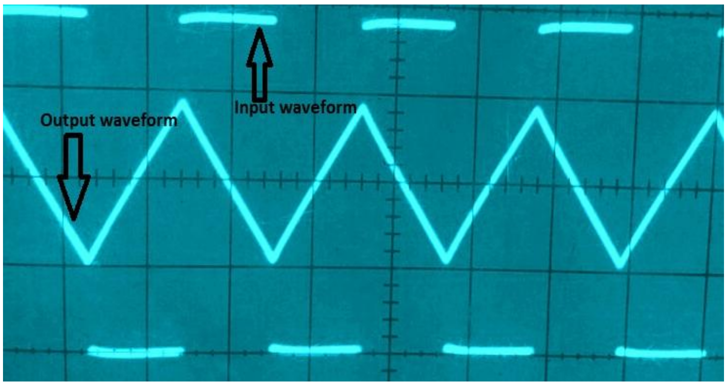

Fig19. *Time response of IOI*

Fig19 shows experimentally obtained input & output waveforms for IOI. Same oscilloscope is being used here as that of FOD.

6. Discussions and Conclusions

From Fig3 it can be said that, -45° phase angle is been exhibited by FOE over the frequency range of 400 Hz to 800 Hz, whereas Fig6 shows that phase plot of FOD exhibits constant phase angle of -45° over frequency range of 500 Hz to 1.4 kHz. Fig14 shows phase plot of FOI gives same response within the frequency band of 30 kHz to 70 kHz.

It is been concluded from above study that, designed RC ladder network works successfully as a Fractional order element (FOE) as it provides phase angle of -45°. And same network if used in any differentiator circuit and integrator circuit in place of capacitor, can impart fractional order to the circuit. By changing the R and C values the frequency range of constant phase can be shifted.

The use of the fractional order differentiator and integrator in different signal conditioning circuits may also be considered as further research area.

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