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Reduction of PAPR and BER by Using Golay Sequences for OFDM System

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Abstract: *PAPR (Peak To Average Power Ratio) is the major drawback in multicarrier systems. In this paper, we proposing a low complexity Golay sequence coder followed by special Fractional Fourier Transform (FRFT) block to reduce the PAPR. The input data encoder which provides low order Golay sequences coded modulation is transmitted through FRFT block and it is designed to provide optimal decorrelation between signal and noise. This provides low complexity, low BER (Bit Error Rate) and PAPR reduction for previous existing schemes and also observes the BER and PAPR results by considering different modulation techniques (i.e., 16-QAM and 64-QAM)*

Keywords: *PAPR, OFDM, BER, QAM (Quadrature- Amplitude Modulation), Golay Sequence and FRFT.*

1. Introduction

Multicarrier techniques transmit data by dividing the stream into several parallel bit streams. Each of the Sub channels has a much lower bit rate and is modulated onto a different carrier. Orthogonal frequency-division multiplexing (OFDM) is one type of multicarrier transmission system. Orthogonal frequency division multiplexing (OFDM) is a method of transmitting data simultaneously over multiple equally spaced carrier frequencies, using Fourier transform processing for modulation and demodulation .The method has been proposed or adopted for many types of radio systems such as wireless local-area networks and digital audio and digital video broadcasting ,OFDM offers many well-documented advantages for multicarrier transmission at high data rates, particularly in mobile applications. In general the OFDM signal is the sum of many independent signals modulated onto sub channels of equal bandwidth. Let us define N symbols in OFDM as $\{X_n, n = 0,1, \dots \dots \dots.., N-1\}$.The complex baseband representation of a multicarrier signal consisting of N subcarriers is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n \Delta f t}; \;\; 0 \le t < NT \tag{1}$$

Where $j = \sqrt{-1}$, Δf is the subcarrier spacing, and NT denotes the useful data block period. In OFDM systems the subcarriers are assumed to be mutually orthogonal.

High peak to average power ratio (PAPR) is the major drawback of multicarrier transmission. Various researches had been proposed to reduce this factor. If the peak transmit power is limited by either regulatory or application constraints, the effect is to reduce the average power allowed under multicarrier transmission relative to that under constant power modulation techniques. This in turn reduces the range of multicarrier transmission. Moreover, to prevent spectral growth of the multicarrier signal in the form of inter- modulation among subcarriers and out-of-band radiation, the transmit power amplifier must be operated in its linear region (i.e., with a large input backoff), where the power conversion is inefficient. This may have a deleterious effect on battery lifetime in mobile applications. In many low-cost applications, the drawback of high PAPR may outweigh all the potential benefits of multicarrier transmission systems.

These techniques are divided in three groups. The first one is based on coding techniques such as Reed Muller code, [3] and block code, [4]. The encoding based PAPR reduction techniques have shown significant benefits for reduced number of subcarriers N (about 8 to 16) since the length and therefore the performances of codes are directly related to N. The second group is based on adding an extra signal to the data such as Clipping [5], Tone reservation [6] and Active Constellation Extension [8]. The failure of this class of techniques is the excessive increase of transmitted power. The third group is based on probabilistic techniques such as Partial transmit Sequences [7], Random Phasor [9] and Selective Mapping [10]. These techniques have shown significant PAPR enhancements at the cost of high complexity and also bit error rate loss due to side information energy. Interesting technique proposed in [11], [12] and [13] is based on the use of Golay sequences as PAPR reduction technique.

In this paper FFT module is replacing by FRFT and it is a linear transform, weak correlation of noise and signal for particular time frequency space. By using FRFT in OFDM system, we can reduce the PAPR without increasing the complexity i.e., it has same complexity as FFT.

This paper we considered special encoder that contains in combining low order Golay coder (which provide Golay sequences), with special FRFT block that offers minimum PAPR without any additive BER loss.

The paper is organized as follows. Section II contains brief overview of Golay sequence, FRFT and QAM (Quadrature- Amplitude Modulation). After that, Section III and IV contain the proposed method and Experimental results and finally, Section V gives the conclusion and future scope.

2. GOLAY SEQUENCES, FRFT AND QAM

2.1. Golay Sequence

Any pair of two sequences is said to be golay sequences verifies that the sum of its apiodic autocorrelation functions is equal to 0. Marcel Golay was introduces Golay sequence, and they became a good method used to reduce PAPR in OFDM system, [11], [12], [13], [19]. Let us first define some parameters, let $a = (a_0, a_1, \ldots\ldots.. a_{n-1})$ a sequence of length n and a characteristic H, such that each entry $a_i \in Z_H$, $\xi = exp(2\pi\sqrt{-1}/H)$, the aperiodic autocorrelation of a in u is given by:

$$C_a(u) = \sum_{i=0}^{n-1-u} \xi^{a_i - a_{i+u}} \tag{2}$$

Any pair of Golay or complementary sequences is a pair (a, b) that verifies that the sum of its autocorrelation functions is equal to 0; $C_a(u) + C_b(u) = 0, 0 < u < n$ called a Golay complementary sequence or Golay sequence. The instantaneous power of the signal a is given by

$$P_a(t) = |s(t)|^2$$

$$= \sum_{i,j} \xi^{a_i(t) - a_j(t) - H(i-j)\Delta ft} \tag{3}$$

Let $a_i(t) = a_i \; for \; 0 \leq \Delta ft \leq 1 \; and \; j = i + u$, then,

We obtain:

$$P_a(t) = n + \sum_{u \neq 0} \sum_i \xi^{a_i - a_u - Hu\Delta ft} \tag{4}$$

By adopting the formula of auto-correlation, the formula of the instantaneous power is given by:

$$P_a(t) = n + \sum_{u \neq 0} \sum_i C_a(u) \xi^{-Hu\Delta ft} \tag{5}$$

Let $a = (a_0, a_1, \ldots\ldots.. a_{n-1})$ and $b = (b_0, b_1, \ldots\ldots.. b_{n-1})$ two code words, x and y are a pair of Golay complementary sequences if $C_a(u) + C_b(u) = 0$ for $u \neq 0$. Using the previous definitions, we can conclude that the PAPR of a Golay sequence is at most equal to 2. The proof is obvious since the sum of instantaneous power is equal to 2n, and given that the power is positive, so each output is less than 2n. Then since the average power is equal to n. so the PAPR is 3dB.

2.1.1. Generation of Golay sequences

Assuming that Golay sequences provides reduced PAPR in OFDM system, the problem is how to generate these sequences. Davis AND Jedwab, in [12], gave a general method for generating Golay sequences using Boolean functions. A Boolean function is a function of $\mathbb{Z}_2^m = \{(x_1, x_2, \dots \dots \dots . x_m) | \{0,1\}\}\ in\ \mathbb{Z}_2$ each variable x_i is a Boolean $f_i(x_1, x_2, \dots \dots \dots . x_m) = x_i$ By considering the 2^m monomial $1, x_1, x_2, \dots \dots \dots . x_m, x_1x_2, x_2x_3, \dots x_{m-1}x_m$

$, \dots \dots, x_1x_2 \dots \dots \dots . x_m$ Thus, any Boolean function is a linear combination in $\mathbb{Z}_2$ of these monomials, such that the coefficients of each monomial belongs to $\mathbb{Z}_2$. Davis and Jadweb have specified a sequence f of length 2^m for each function f by a list of values given by $f_i(x_1, x_2, \dots \dots \dots . x_m)$ as $(x_1, x_2, \dots \dots \dots . x_m)$ where all 2^m values are in lexicographic order . So, if $(i_1, i_2, \dots \dots \dots . i_m)$ is the binary representation of $i = \sum_{j=1}^{m} i_j 2^{mj}$, then i^th item **f** is $f_i(i_1, i_2, \dots \dots \dots . i_m)$. Using these boolean functions, Davis and Jadweb determine an explicit form for generating a Golay sequence, such that for any permutation π of symbols (1,2,….m) and any $C, C_k \in \mathbb{Z}_{2^h}$

$$a(x_1, x_2, \dots \dots \dots . x_m) = 2^{\mathrm{h}-1} \sum_{k=1}^{m-1} x_{\pi(k)}\, x_{\pi(k+1)} + \sum_{k=1}^{m} c_k\, x_k \ + c \qquad (6)$$

is a Golay sequence in $\mathbb{Z}_{2^h}$ of length 2^m. This definition of Golay sequences gives $2^{\mathrm{h(m+1)}}$. m!/2 Golay possible sequences in $\mathbb{Z}_{2^h}$ of length 2^m.

2.2. Fractional Fourier Transform

The usual interpretation of the Fourier transform is as a transformation of a time domain signal into a frequency domain signal. On the other hand, the interpretation of the inverse Fourier transform is as a transformation of a frequency domain signal into a time domain signal. Apparently, fractional Fourier transforms can transform a signal (either in the time domain or frequency domain) into the domain between time and frequency: it is a rotation in the time-frequency domain.The fractional Fourier Transform is a generalization of the Fourier Transform [18].

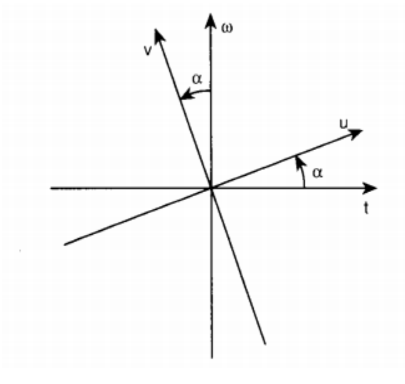

Fig1. *Time-frequency plane Fractional Fourier Transform.*

The FRFT of a signal $s(t)$ is defined as follows:$F_\alpha(s) = S_\alpha(u) = \int_{-\infty}^{+\infty} s(t) K_\alpha(t, u) dt$. Where a is a real number known as FRFT order, $\alpha = a\frac{\pi}{2}$ is the angle of FRFT, and $K_\alpha(t, u)$ is the kernel of FRFT

$$= \begin{cases} \sqrt{\frac{1 - jcot(\alpha)}{2\pi}} \exp\left(j\frac{t^2 + u^2}{2}\cot(\alpha) - jutcsc(\alpha)\right. & \alpha \neq n\pi \\ \delta(t - u) & \alpha = 2n\pi \\ \delta(t + u) & \alpha + \pi = 2n\pi \end{cases} \qquad (7)$$

The FRFT can be considered as a projection of the signal on an axis which forms an angle α with the time axis. From the definition above, for $\alpha = 0$, there will be no change after applying fractional Fourier transform, and for $\alpha = \pi/2$, fractional Fourier transform becomes a Fourier transform, which rotates the time frequency distribution with $\pi/2$. For other value of α, fractional Fourier transform rotates the time frequency distribution according to α. Figure shows the Time-frequency plane Fractional Fourier Transform.

The FRFT gives great satisfactions in many signals processing applications such optical communications, signal filtering and also beam forming for fading channels [17].

Multicarrier modulation that uses traditional Fourier Transform attempts a frequency windowing of bandwidth. The effect of the time-invariant channel distortions can be compensated for by sub channel by sub channel basis single tap frequency domain equalizers. Consequently, the overall traditional multicarrier system can be seen as an optimal Fourier-domain filter. However, if the channel is time-varying, the traditional multicarrier system loses optimality since optimal recovery operator is generally time-variant. This means that it cannot be implemented in the conventional Fourier domain and is exactly the reason that motivates the use of an FRFT-based technique.

2.3. Quadrature Amplitude Modulation

QAM (quadrature amplitude modulation) is a method of combining two amplitude-modulated (AM) signals into a single channel, thereby doubling the effective bandwidth. In a QAM signal, there are two carriers, each having the same frequency but differing in phase by 90 degrees (one quarter of a cycle, from which the term quadrature arises). One signal is called the I signal, and the other is called the Q signal. Mathematically, one of the signals can be represented by a sine wave, and the other by a cosine wave. The two modulated carriers are combined at the source for transmission. At the destination, the carriers are separated, the data is extracted from each, and then the data is combined into the original modulating information.

When using QAM [20], the constellation points are normally arranged in a square grid with equal vertical and horizontal spacing and as a result the most common forms of QAM use a constellation with the number of points equal to a power of 2 i.e. 4, 16, 64 . . . . By using higher order modulation formats, i.e. more points on the constellation, it is possible to transmit more bits per symbol. However the points are closer together and they are therefore more susceptible to noise and data errors.

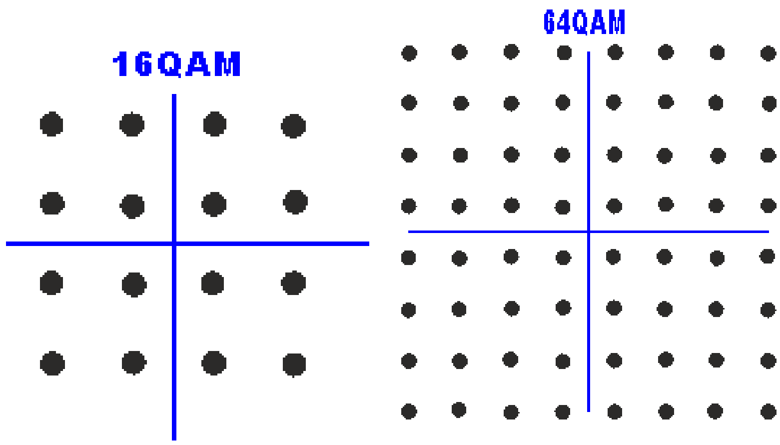

Fig2. *QAM (16-bit & 64-bit Quadrature amplitude Modulation)*

3. Proposed Method

In this section, the proposed method is presented. Firstly, we study the binary case (i.e. h = 1) and m = 2. So, we obtain 8 binary codewords of length 4. Using the formula given by Davis and Jedwab in [12], the 8 Golay sequences are 0001, 0010, 0100, 0111, 1000, 1011, 1101 et 1110. We present in this figure the fractional Fourier Transform (FRFT) used for reducing PAPR of the OFDM systems, [14]. The proposed method is given by figure 3. Computer simulation of bit error rate values the effectiveness of coder implemented for different modulators.

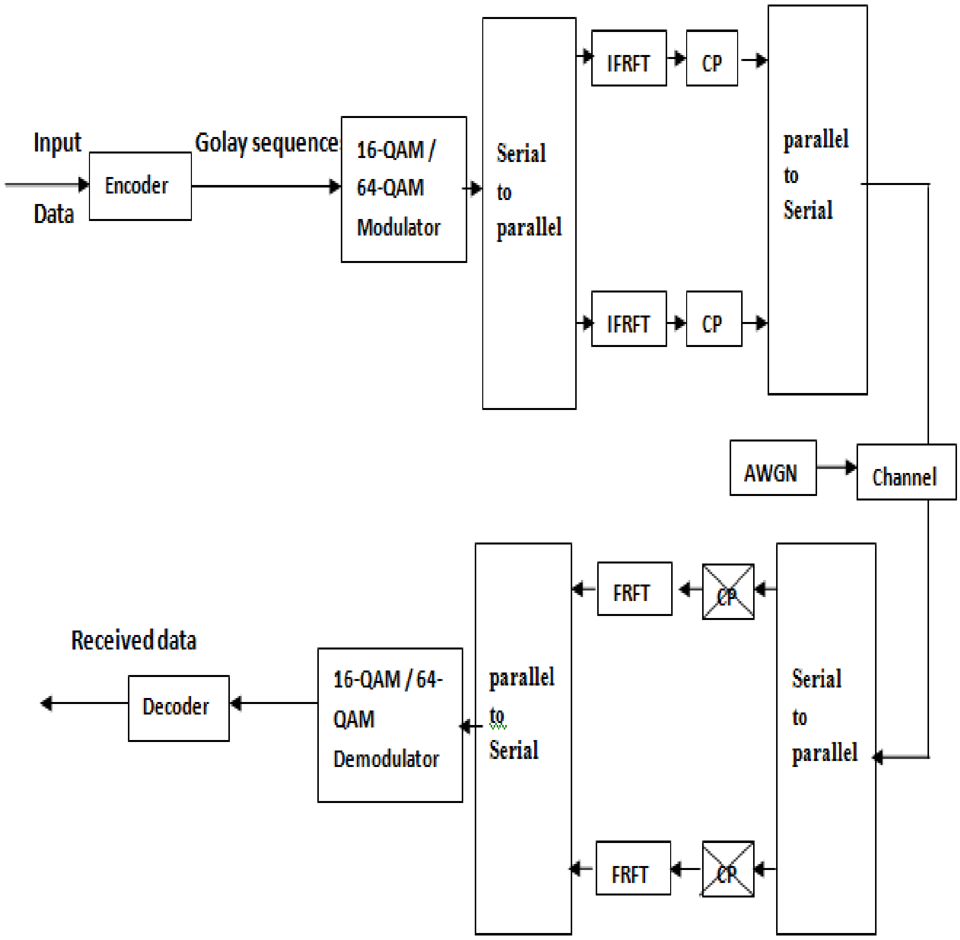

Fig3. *Proposed Method*

4. EXPERIMENTAL RESULTS

To evaluate the performance analysis of this method by using simulator the following parameters are considered and listed in table I.

Table1. *The parameters for simulation*

Modulation	16-QAM,64-QAM
Number of sub carriers	52
Number of FFT points	64
Channel model	Channel with AWGN
Coding rate	2/3
Decoding	Soft Viterbi Algorithm

PAPR : it is defined by

$$PAPR = \frac{max|x(t)|^2}{E|x(t)|^2} \quad (8)$$

Where E(.) denotes expectation. The complementary cumulative distribution function (CCDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold. The cumulative distribution function (CDF) of the amplitude of a signal sample is given by, [1]:

$$F(z)=1-\exp(z) \quad (9)$$

BER: it is the ratio of difference between transmitted and received data to length of the data.

Fig 4 shows bit error rate of the OFDM system using coded and uncoded system. This figure shows Golay sequences coded system containing low bit error rate(BER) compared to non Golay sequences in 16-QAM (Quadrature Amplitude Modulation).

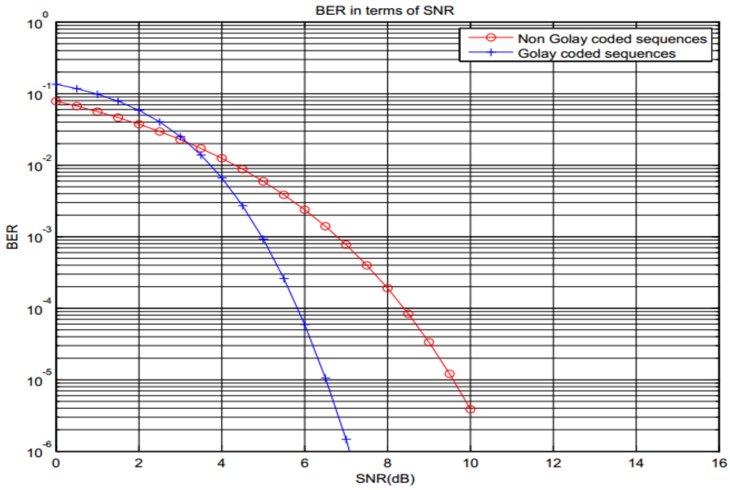

Fig4. *BER in terms of both Golay coded sequences and non Golay coded sequences in 16-QAM.*

Fig 5 gives the importance of Fractional Fourier Transform over conventional Fourier transform in terms of PAPR. We clearly notice that the PAPR for the $CCDF = 10^{-3}$, for FRFT it is 4.9dB and For FFT it is 8.8dB. which gives the better performance combination of Golay sequences and Fractional Fourier Transform.

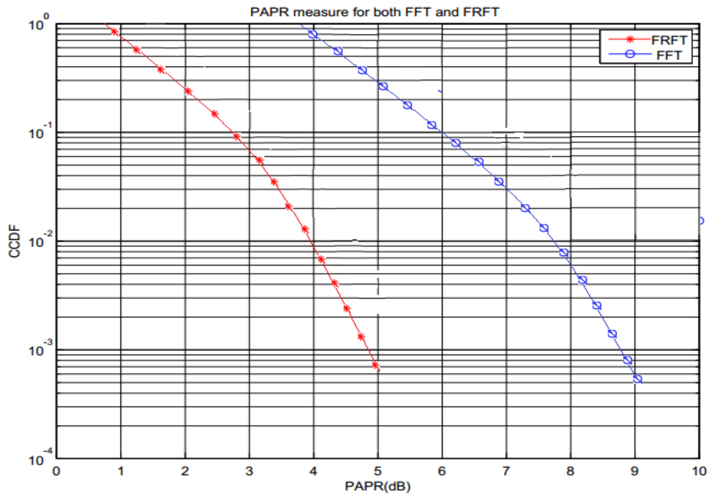

Fig5. *PAPR measure for both FFT and FRFT*

Figure 6 shows the influence of various FRFT on the PAPR. when he angel is close to 1, the PAPR tends towards that of conventional Fourier transform.

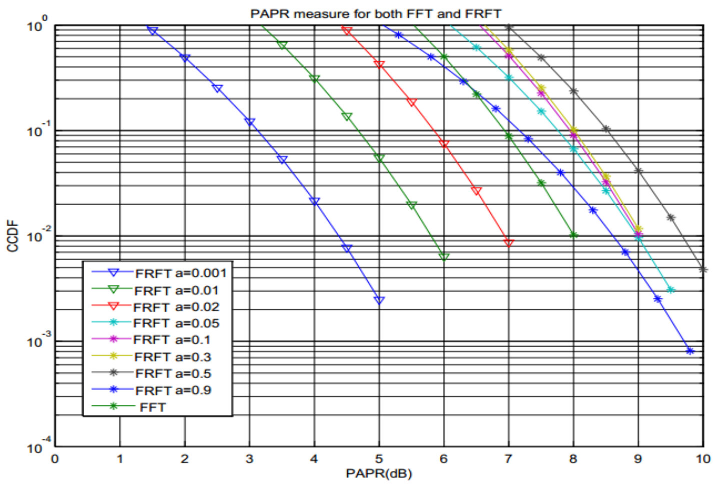

Fig6. *Optimization of FRFT angle of minimum PAPR*

Figure 7 shows the BER comparison of uncoded Golay system (16-bit QAM), Golay coded system(16-bit QAM) and coded Golay sequences(64-bit QAM).

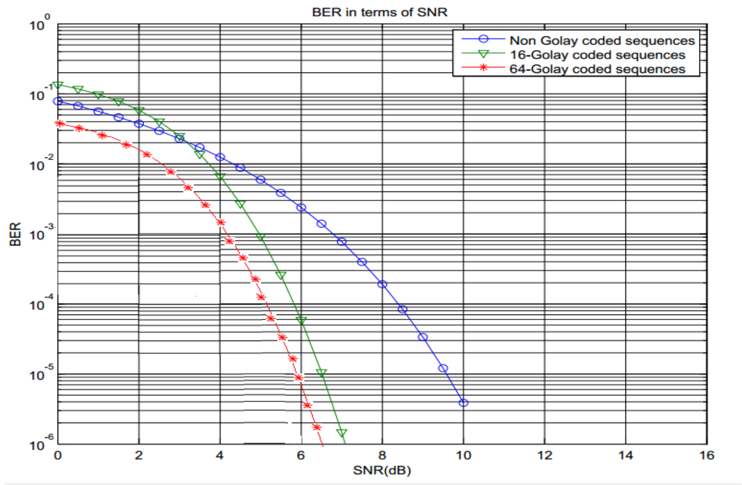

Fig7. *BER in terms of SNR for 16-bit QAM(both coded , uncoded- Golay sequences) and 64-bit QAM.*

5. Conclusion and Future Work

Here, proposes a new technique that uses Golay sequences in conjunction with Fractional Fourier Transform for reducing PAPR and BER. The main advantage is to exploit the well known efficiency of both Golay and FRFT in reducing PAPR and maintaining low complexity in OFDM system. Finally, analyzing the results for 16-QAM and 64-QAM. Furthermore, analyze the BER and PAPR results for Different equalizer's (differential equalizers, adaptive equalizers etc) and different modulation techniques (128-QAM etc).

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