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A LFM Interference Suppression Scheme Based on FRFT and Subspace Projection

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ABSTRACT

The linear frequency modulated (LFM) interference is becoming more and more serious in radar, GPS and other communication fields. Consider the good time-frequency aggregativeness of Fractional Fourier Transform (FRFT) and favourable anti-jamming performance of subspace projection interference suppression (SPIS), this paper proposes a LFM Interference Suppression Scheme Based on FRFT and Subspace Projection. The simulation result proves the proposed scheme can achieve good LFM interference suppressing performance.

Keywords: linear frequency modulated signal, SPIS, FRFT

INTRODUCTION

Nowadays, GPS is extremely useful in many military and civil areas. However, in real applications, GPS signals are often disturbed by various kinds of interferences. The minimum power for GPS signals is -163dBW when reaching the ground which made GPS signals vulnerable [1]. Thus, the main challenge in receiving GPS signals is the suppression of interferences.

In recent years, the effect caused by nonstationary interferences on GPS has become a hotspot. As a typical nonstationary interference, linear frequency modulated (LFM) signals have the characteristic of time-varying frequency [2]. Normally, the jamming effect caused by LFM interference is serious than that caused by stationary interference [2]. However, Fourier Transform (FT) can hardly deal with nonstationary interference, especially LFM interference. In order to fix this problem, numbers of time-frequency analysis methods are under study, for instance Winer-Ville distribution [3] (WVD), Short-Time Fourier Transform [4] (STFT) and Fractional Fourier Transform (FRFT). Due to the existence of cross-term in WVD and the selection of window function in STFT [5], both time-frequency analysis methods have encountered their disadvantages. In contrast, FRFT has lower complexity as well as taking fully advantage of the time-frequency aggregativeness.

On the other hand, space-time adaptive processing (STAP) is also an important anti-jamming method [6]. STAP can be achieved in many ways and subspace projection interference suppression (SPIS) is one of them [7]. This paper proposes a LFM interference suppression scheme based on FRFT and an improved SPIS algorithm. The scheme can take advantage of both time-frequency synthesis and space-time synthesis. The simulation result proves the proposed scheme can achieve good LFM interference suppressing performance.

THE PRINCIPLE OF FRFT

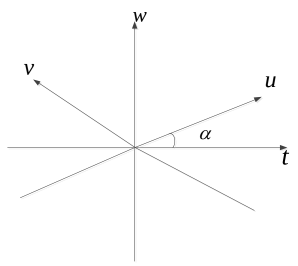

Figure1. *The principle of FRFT*

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In 1980, Namias proposed a disparate Fourier transform form, namely FRFT [8]. The FRFT of a signal can be considered as an anticlockwise rotation from the time axis to the μ axis with an rotation angle α . Normally, the μ axis is called the fractional Fourier domain. The principle can be clearly expressed in Fig.1.

The FRFT of a signal $x(t)$ can be defined as

$$X_p(u) = \int_{-\infty}^{+\infty} x(t) K_p(u,t) dt \tag{1}$$

$$K_p(u,t) = \begin{cases} \sqrt{1 - j\cot\alpha e^{j\pi[(\mu^2+t^2)\cot\alpha - 2ut\csc\alpha]}}, \alpha \neq n\pi \\ \delta(t-u), \alpha = 2n\pi \\ \delta(t+u), \alpha = (2n+1)\pi \end{cases} \tag{2}$$

where $K_p(u,t)$ is the core function. p is the order of the transform and $\alpha = p\pi/2$. Note that if $p=1$, namely $\alpha = \pi/2$, the FRFT reduces to conventional Fourier transform. A typical LFM signal can be expressed as

$$j(t) = e^{j2\pi(f_0 t + \frac{\mu_0 t}{2})} \tag{3}$$

where f_0 is the initial frequency and μ_0 is the sweep rate. The most important factor for FRFT to eliminate the LFM interference is to estimate f_0 and μ_0 . Whenever f_0 and μ_0 is estimated, the LFM interference can be reconstructed.

SUBSPACE PROJECTION

The Basic Principle of Subspace Projection

Consider the case of GPS signal corrupted by LFM interference and noise. The received signal can be written as

$$X(n) = S(n) + I(n) + N(n) \tag{4}$$

$S(n)$, $I(n)$ and $N(n)$ stands for GPS signal, interference and noise, respectively. Normally, GPS signal, interference and noise are independent from each other. Thus, the covariance matrix of the received signal is

$$R_X = E\left\{X(n)X(n)^H\right\} = R_S + R_I + R_N \tag{5}$$

Where R_S , R_I , R_N stands for the covariance matrix of GPS signal, interference and noise. The definitions of these three parameters are as follows:

$$\begin{cases} R_S = E\left[S(n)S(n)^H\right] \\ R_I = E\left[I(n)I(n)^H\right] \\ R_N = E\left[N(n)N(n)^H\right] \end{cases} \tag{6}$$

Meanwhile, $R_N = \sigma_n^2 I$, σ_n^2 is the variance of the noise, I is a unit matrix with the order of M . Because

R_X is a positive definite matrix, the eigenvalue of R_X can be written as:

$$\lambda_1 > \lambda_2 > \ldots > \lambda_r > \lambda_{r+1} = \ldots = \lambda_n \tag{7}$$

The subspace of the interference and signal is $Q_{SN} = [q_1, q_2, \ldots, q_n]$ and the subspace of the interference is $Q_I = [q_{n+1}, q_{n+2}, \ldots, q_M]$. The orthogonal complementary space $Q_I^{\perp}$ can be deduced by the following formula

$$Q_I^{\perp} = I - Q_I Q_I^{\perp} \tag{8}$$

Then the received signal is projected to $Q_I^{\perp}$. Thus the output can be written as

$$Y(n) = Q_I^{\perp} X(n) = Q_I^{\perp}(X_S(n) + N(n)) \tag{9}$$

Theoretically, according to the formula (9), the interference is absolutely eliminated.

Weighting Processing

Due to the vast interferences, the fully elimination of the interference can't be realized. We propose a weighting operation to dispose the output $Y(n)$.

Suppose the weight coefficient is $W_M \subset C^{M\times 1}$. After weighting operation, the output signal can be written as:

$$Z_n = W_M^H Y(n) = W^H Q_I^{\perp} X(n) = W^H Q_I^{\perp}(X_S(n) + N(n)) \tag{10}$$

The choose of W_M can be gained according to MSINR criterion.

$$W_M = \arg\max \frac{E\left|W^H Q_I^{\perp} X_S(n)\right|^2}{E\left|W^H Q_I^{\perp} N(n)\right|^2} \approx \arg\max \frac{E\left|W^H Q_I^{\perp} Y(n)\right|^2}{E\left|W^H Q_I^{\perp} N(n)\right|^2} \tag{11}$$

Let $s(k)$ and S stand for GPS signal and its space-time direction vector. The Spread spectrum sequence cycle $L = 1023$. Define $q = [s(1), s(2), ..., s(L)]^T \otimes S$, after dispreading,

$$Z = q^H Y(n) = z_s + z_n \tag{12}$$

The SINR can be defined as

$$SINR = \frac{E^2(z_s)}{E(z_n^{\ 2})} = \frac{LM\left(1 - \sum_{i=1}^{P} |\alpha_i|^2 |\beta_i|^2\right)}{\sigma^2} \tag{13}$$

α_i and β_i stand for the correlation coefficient of the space-time domain for the i inference. Meanwhile, a series GPS data includes 20 Spread spectrum cycle, thus

$$E(z_n) = \frac{\sum_{n=1}^{N} z_n}{N} \tag{14}$$

Thus, Substituting (14) into (13), we obtain formula as follows

$$SINR = \frac{20LM}{\sigma^2} - \frac{LM\sum_{i=1}^{P}\left(|\alpha_i|^2 \sum_{n=1}^{20} |\beta_{ni}|^2\right)}{\sigma^2} \tag{15}$$

Where $20LM/\sigma^2$ stands for the SINR under ideal condition, $LM \sum_{i=1}^{P}\left(\left|\alpha_i\right|^2 \sum_{n=1}^{20}\left|\beta_{ni}\right|^2\right)/\sigma^2$ stands for the data loss during the processing. After weighting processing, the output SINR turns to be:

$$SINR = W_M^H \left[\frac{20LM}{\sigma^2} - \frac{LM\sum_{i=1}^{P}\left(|\alpha_i|^2 \sum_{n=1}^{20} |\beta_{ni}|^2\right)}{\sigma^2}\right] \tag{16}$$

Compare formula (16) to formula (15), the weighting processing helps to improve $\left(10\log W_M^H M\right)$ dB. Meanwhile the traditional subspace projection only improves $\left(10\log M\right)$ dB.

Joint Interference Suppression

According to the FRFT and weighting subspace projection, we propose a joint interference suppression method. Because FRFT has good energy aggregation performance for LFM interference and weighting subspace projection method can spread the array elements, the proposed method takes advantage of both ways.

The schematic diagram of joint interference suppression method is shown in Fig. (2)

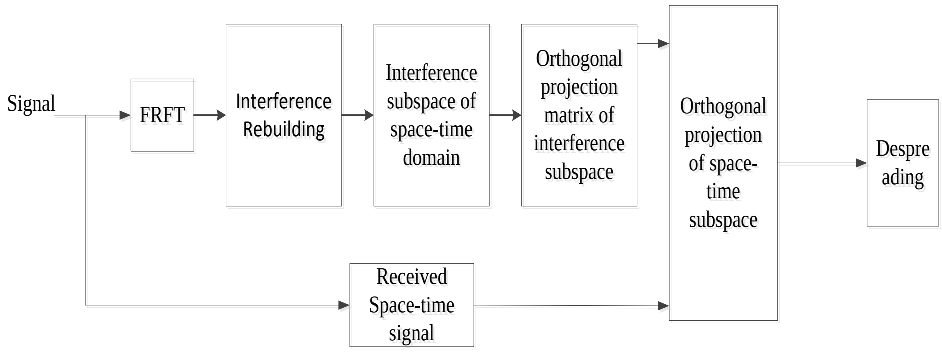

Figure2. *Schematic diagram of joint interference suppression*

Thus, the proposed joint interference suppression method is summarized as follows

Step1: Take α as the independent variable, and use it to conduct Continuous fractional Fourier transform so that $(\alpha\ ,\ u)$ based two-dimensional surface can be formed. Then, use least mean square criterion to lock the point that makes the interference energy filtered out possible.

Step2: According to the specific point $(\alpha_0\ ,\ u_0)$, estimate the lth original frequency and frequency modulation index, and then reconstruct the lth signal.

Step3: Construct the Space-time interference steering vector of the lth signal based on step2.

Step4: Repeat step2 and step3 until the Space-time interference subspace is obtained.

Step5: Build the Orthogonal projection matrix, and project the space-time received signal into the orthogonal subspace of the interference subspace to obtain the interference suppression signal.

Step6: Dispread the interference suppression signal, and after judgment the desired GPS signal is obtained free of interferes.

DIGITAL SIMMULATION

To confirm the analysis and gain more insight into the achievable performance, we provide numerical simulation results in this section. In our simulation, an uniform line antenna (ULA) with M=50 elements spaced a half wavelength is considered. The LFM interfering source is assumed to have a DOA of -60°, with corresponding interference-to-noise ratio (INR) is 50dB. The presumed desired GPS signal impinges on the array from direction $\theta_0=20^\circ$. In the examples, the sample covariance matrix is computed based on $N=512$ data snapshots, and all of the following results are calculated based on 300 Monte Carlo experiments.

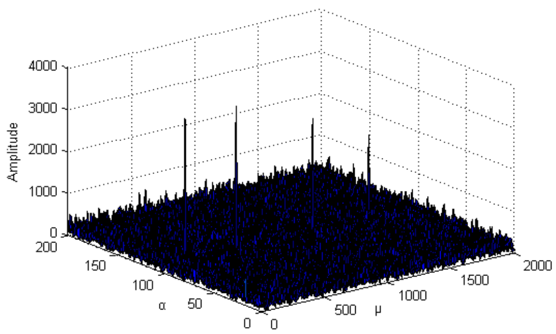

Figure3. *The GPS signal capture result before processing*

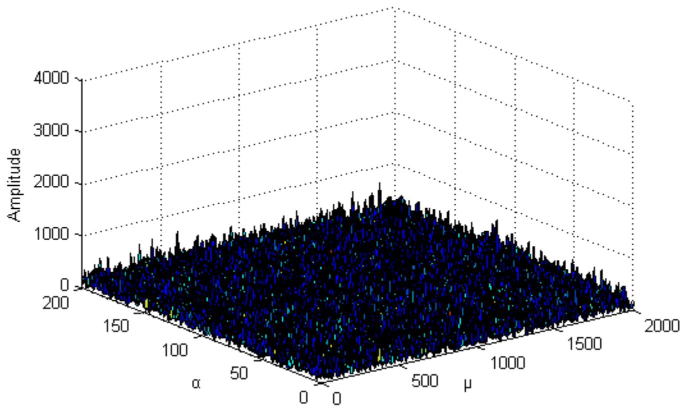

Figure4. *The GPS signal capture result after processing*

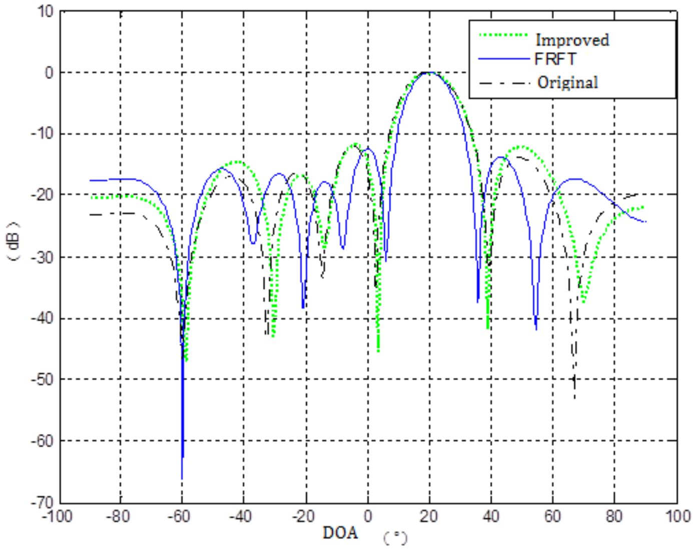

Figure5. *The beam pattern comparisons*

In the example, the SNR is set to be a constant of -20dB. Fig.3 is the capture result of the received signal before the process. And Fig.4 is the corresponding capture result after interference suppression. It can be seen that obvious peaks occur in Fig.3. This is because the LFM interferes are present in the receiver. While the signal amplitude distribution is relatively flat in Fig.4 due to the LFM interference suppressed.

Fig.5 shows that the proposed FRFT based method and the joint subspace projection algorithm form deeper nulls in the interference direction of -60° than that of the original method. Moreover, with the two proposed methods, the main lobe of the beam pattern becomes Narrower and the side lobe is much lower. Thus, the proposed methods have much better beam directivity. In addition, the proposed FRFT based method is slightly better than the joint subspace projection algorithm. Unfortunately, the original method has a poor performance.

In addition to the beam pattern analysis above, we conduct quantitative analysis mainly from the perspective of mathematical statistics. In the simulation, the SNR varies form -15dB to 25dB.

CONCLUSION

In this work, a LFM Interference Suppression Scheme Based on FRFT and Subspace Projection is developed. In the proposed method, the improved subspace is combined with FRFT to derive the joint LFM interference suppression algorithm. Digital simulation results prove the proposed scheme can achieve good LFM interference suppressing performance and outperforms the other LFM interference suppressing methods.

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