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Calculation Method for Predicting the Insertion-Loss of Soundproofing Ventilation Unit

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ABSTRACT

A previous study discussed a rectangular soundproofing ventilation unit (SVU) having an inlet and an outlet located on the opposite face. However, as a result this enables only a small outlet are to be constructed, which causes ventilation features to suffer. To improve this problem, a rectangular SUV having an inlet and an outlet which is located on the crossed right angle face is proposed in this work. The calculation method is proposed to determine the insertion-loss of this SVU by solving the wave equation including a higher-order mode effect.

Keywords: Wave equation, sound pressure, resonance frequency, higher-order mode

INTRODUCTION

In previous paper, the authors are presented a concept for manufacturing windows called Soundproofing Casement Windows which are capable of ventilating, regulating sunlight and reducing traffic noise inside the homes of developing countries in tropical climate zones[1] . A previous study discussed a rectangular soundproofing ventilation unit (SVU) having an inlet and an outlet located on the opposite face. However, as a result this enables only a small outlet are to be constructed, which causes ventilation features to suffer. In order to improve this problem, a rectangular SUV having an inlet and an outlet which is located on the crossed right angle face is proposed [2]. Due to the fact that SVU must have a large volume to attenuate the low frequency range of noise outside, sound propagating through SVU is a combination of a plane wave and the higher order mode wave. It generates very high levels of sound pressure, especially at resonance frequency. Therefore, it is necessary to reduce sound pressure levels or prevent the generation of higher order mode waves in order to maximize the soundproof capability

Actually, the acoustic performance of an acoustic filter is evaluated using either the transmission loss or the insertion loss, while insertion loss values are preferred for most applications [3]. This paper will focus on the calculation of insertion loss of SVU for constant velocity volume noise source. The computation was based on solving the wave equation considering the higher order mode effects.

METHOD OF ANALYSIS

Insertion-Loss

Let U_r and R_r are the constant volume velocity and radiation impedance of the noise source as shown in Figure 1(a). The radiated power at one point from the noise source is defined as [4]

$$W_r = |U_r|^2 R_r \tag{1}$$

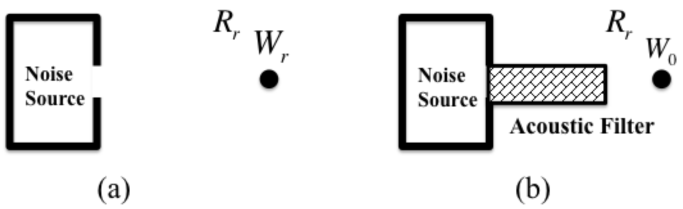

Figure1. *Define of Insertion-Loss*

When the acoustic filter is inserted between that point and the noise source as shown in Figure 1(b), by the reason of the radiated impedance R_r is very small, following equation is obtained

$$U_r = D\,U_0 \tag{2}$$

where U_0 and $D$are the volume velocity at the outlet and the four-pole parameter D of the filter. Therefore, radiated power from the outlet becomes

$$W_0 = \left|U_0\right|^2 R_r = \left|\frac{U_r}{D}\right|^2 R_r \tag{3}$$

and the insertion-loss IL is given by following

$$IL = 10\,Log_{10}\frac{W_r}{W_0} = 20 Log_{10}\left|D\right| \tag{4}$$

Note that, the four-poles parameter is defined as

$$\begin{vmatrix} P_i \\ U_i \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \begin{vmatrix} P_0 \\ U_0 \end{vmatrix} \tag{5}$$

here $P_i$and $U_i$are the sound pressure and velocity at the input , P_0 and U_0 are those of the output. The D parameter is given by

$$D = \left.\frac{U_i}{U_0}\right|_{P_0=0} \tag{6}$$

Calculate the Insertion-Loss of SVU

Model of the rectangular soundproofing ventilation unit which has a dimension of $a´\ b´\ d$ is shown in Figure 2. Dimension of an input and output are $(a_{i2} - a_{i1}) \times (b_{i2} - b_{i1})$ and $(a_{02} - a_{01}) \times (d_{02} - d_{01})$, they located on the face which has a section area of $S_{ab} = a´\ b$and $S_{ad} = a \times d$, respectively.
Wave equation in terms of velocity potential F is given by

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi \tag{7}$$

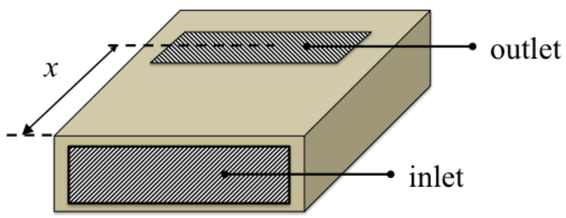

Figure2. *Soundproofing ventilation unit (SVU)*

where c is the sound velocity. Let $\Phi = \sqrt{2}\,\phi\,\exp(j\omega t)$ ($j^2 = -1$, $\omega = kc$, k : wave number) then the general solution of (7) can be given as

$$\phi = \left(Ae^{\mu z} + Be^{-\mu z}\right)\left(C\sin(\alpha x) + D\cos(\alpha x)\right)\left(E\sin\left(\sqrt{s^2-\alpha^2}\,y\right) + F\cos\left(\sqrt{s^2-\alpha^2}\,y\right)\right) \tag{8}$$

where $\mu^2 = s^2 - k^2$, A, B, C, D, E and F are arbitrary constants determinable from the boundary conditions, other symbols are constants.

Let $-\partial\phi/\partial x$, $-\partial\phi/\partial y$ and $-\partial\phi/\partial z$ are the velocity component in the x , y and z directions, respectively. Assuming the walls of the cavity to be perfectly rigid and the loss at the wall can be neglected then boundary conditions are :

From Figure 2, at the left and right side of the unit

[1] at $x = 0$, $-\partial\phi/\partial x = 0$ (9)

[2] at $x = a$, $-\partial\phi/\partial x = 0$ (10)

At the bottom and top side

[3] at $y = 0$, $-\partial\phi/\partial y = 0$ (11)

[4] at $y = b$, $-\partial\phi/\partial y = V_0\, F_0(x,z)$ (12)

At the front and back side

[5] at $z = 0$, $-\partial\phi/\partial z = V_i\, F_i(x,y)$ (13)

[6] at $z = d$, $-\partial\phi/\partial z = 0$ (14)

where V_i and V_0 are the driving velocity at the input and output, $F_i(x,y)$ and $F_0(x,z)$ are define as 1 on the area of inlet and outlet, zero at the other area of inlet and outlet, respectively.

Based on the boundary conditions (9) until (14), the average sound pressure of the outlet is calculated as following

$$\bar{P}_0 = j4kZ_0 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[\frac{U_i}{S_{ab}S_i}\frac{I_{m,n}\cos(n\pi)}{\mu_{m,n}\sinh(\mu_{m,n}d)}\Theta_{m,n} + \frac{U_0}{S_{ad}S_0}\frac{O_{m,n}^2}{\beta_{m,n}\tan(\beta_{m,n}d)}\right] \tag{15}$$

where

$$\Theta_{m,n} = \int_{a_{01}}^{a_{02}} \cos\left(\frac{m\pi x}{a}\right)dx \int_{d_{01}}^{d_{02}} \cosh\left(\mu_{m,n}(z-d)\right)dz \tag{16}$$

$$\mu_{m,n} = \sqrt{(m\pi/a)^2 + (n\pi/b)^2 - k^2} \tag{17}$$

$$I_{m,n} = \int_{a_{i1}}^{a_{i2}} \cos\left(\frac{m\pi x}{a}\right)dx \int_{b_{i1}}^{b_{i2}}\left(\frac{n\pi y}{b}\right)dy \tag{18}$$

$$\beta_{m,n} = \sqrt{k^2 + \left(\frac{n\pi}{d}\right)^2 - \left(\frac{m\pi}{a}\right)^2} \tag{19}$$

$$O_{m,n} = \int_{a_{01}}^{a_{02}} \cos\left(\frac{m\pi x}{a}\right)dx \int_{d_{01}}^{d_{02}} \cosh\left(\frac{n\pi}{d}(z-d)\right)dz \tag{20}$$

$U_i = V_i S_i$ is an volume velocity supplied from the input and $U_0 = V_0 S_0$ is those of the output,

$Z_0 = \rho c / S_0$ is the characteristic impedance of the output.

When m=0 and n=0, $\beta_{m,n} \to k$, $\mu_{m,n} \to jk$, $I_{m,n} \to S_i$, $O_{m,n} \to S_0$, $\Theta_{m,n} \to S_0$. Therefore, expanding (15) with m=0 and n=0 lead to

$$\bar{P}_0 = j4Z_0 \left[\frac{1}{\sin(kd)} \left(-\frac{S_i}{S_{ab}} U_i + \frac{S_0}{S_{ad}} \cos(kd) U_0 \right) \right. \tag{21}$$

where

$$\Theta_{0,0} = \frac{\sin(d-d_{01})k - \sin(d-d_{02})k}{k} \tag{22}$$

the symbolic $\dot{\sum}$ means $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}$ without m=n=0.

When the center of the outlet locate at $z = d/2$ (16) becomes

$$\Theta_{m,n} = \int_{a/2-\Delta a}^{a/2+\Delta a} \cos\left(\frac{m\pi x}{a}\right) dx \int_{d/2-\Delta d}^{d/2+\Delta d} \cosh\left(\mu_{m,n}(z-d)\right) dz$$

$$= \frac{4a}{m\mu_{m,n}\pi} \cos\left(\frac{m\pi}{2}\right) \sin\left(\frac{m\Delta a\pi}{a}\right) \cosh\left(\frac{\mu_{m,n}d}{2}\right) \sinh\left(\mu_{m,n}\Delta d\right) \tag{23}$$

and Eq.(50) becomes

$$\Theta_{0,0} = \frac{4\Delta a \sin(k\Delta d)}{k} \cos\left(\frac{kd}{2}\right) \tag{24}$$

Substituting Eq. (23) and Eq. (24) into Eq. (21), the outlet pressure can be obtained as

$$\bar{P}_0 = j4Z_0 \left[\frac{1}{\sin(kd/2)} \left(-\tilde{\Theta}_{0,0} U_i + \frac{S_0}{S_{ad}} \frac{\cos(kd)}{\cos(kd/2)} U_0 \right) \right.$$

$$+ \frac{1}{k} \dot{\sum} \left(\frac{I_{m,n}\tilde{\Theta}_{m,n}}{S_{ab}S_i} \frac{\cos(n\pi)}{\mu_{m,n}\sinh(\mu_{m,n}d/2)} U_i + \frac{\tilde{\Theta}^2_{m,n}}{S_{ad}S_0} \frac{1}{\beta_{m,n}\tan(\beta_{m,n}d)} U_0 \right) \tag{25}$$

where

$$\tilde{\Theta}_{0,0} = \frac{2\Delta a \sin(k\Delta d)}{k\, S_{ab}} \tag{26}$$

$$\tilde{\Theta}_{m,n} = \frac{2a}{m\pi} \cos\left(\frac{m\pi}{2}\right) \sin\left(\frac{n\pi d}{a}\right) \sinh\left(\mu_{m,n}\Delta d\right) \tag{27}$$

From (22) the four-poles parameter D as defined in (6) is found as

$$D = \left.\frac{U_i}{U_0}\right|_{P_0=0} = -\frac{S_0}{S_{ad}} \frac{1}{\tilde{\Theta}_{0,0}} \frac{\cos(kd)}{\cos(kd/2)}$$

$$+ \dot{\sum} \left(\frac{S_{ab}S_i}{S_{ad}S_0} \frac{\tilde{\Theta}_{m,n}}{I_{m,n}} \frac{1}{\beta_{m,n}\tan(\beta_{m,n}d)} \frac{\mu_{m,n}\sinh(\mu_{m,n}d/2)}{\cos(n\pi)} \right) \tag{28}$$

RESULTS AND DISCUSSION

The average of outlet sound pressure is derived from (21) in which the first term in the bracket represents the plane wave and the second represents the higher order mode wave. In order to obtain a great insertion loss, we have to make clear the cause and the generation mechanism of those waves in the SVU.

First, with respect to the plane wave, from (21) the sound pressure level is defined as:

$$\bar{P}_{01} = \frac{1}{\sin(kd)}\left(-\Theta_{0,0}U_i + \frac{S_0}{S_{ad}}\cos(kd)U_0\right) \tag{29}$$

$\bar{P}_{01}$ become great at the frequencies where the denominator $\sin(kd)$ become zero. Namely, when $\sin(kd)=0$, these resonance frequencies are

$$kd = \eta\pi \quad \therefore \quad f = \eta\frac{c}{2d} \quad (\eta = 1, 2, \ldots.) \tag{30}$$

Similarly, with respect to higher order mode waves, the sound pressure level is defined as

$$\bar{P}_{02} = \frac{1}{k}\sum^{*}\left(\frac{I_{m,n}\Theta_{m,n}}{S_{ab}S_i}\frac{\cos(n\pi)}{\mu_{m,n}\sinh(\mu_{m,n}d)}U_i + \frac{O^2_{m,n}}{S_{ad}S_0}\frac{1}{\beta_{m,n}\tan(\beta_{m,n}d)}U_0\right) \tag{31}$$

$\bar{P}_{02}$ become great at the frequencies given as follows:

$$\mu_{m,n}\sinh(\mu_{m,n}d) = 0 \quad \therefore \quad f^{(1)}_{m,n} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\eta\pi}{d}\right)^2} \quad (\eta = 0, 1, 2, \ldots.) \tag{32}$$

$$\tan(\beta_{m,n}d) = 0 \quad \therefore \quad f^{(2)}_{m,n} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{d}\right)^2 + \left(\frac{\eta\pi}{d}\right)^2} \quad (\eta = 0, 1, 2, \ldots.) \tag{33}$$

A calculation of insertion loss based on equation (28) is shown in Figure 3 where the SVU has a dimension of *a*=0.3m, *b*=0.1m and *d*=0.6m, respectively. A black dotted line shown the IL of plane wave, the first term in the bracket of (28). The red line shows those of plane wave and higher order mode (2, 0). As is evident from the figure, insertion loss have a minus value at the resonance frequencies which occur corresponding to equation (30) and (32). Note that, our calculation was performed with the outlet located at the center of SVU, when $z = d/2$.

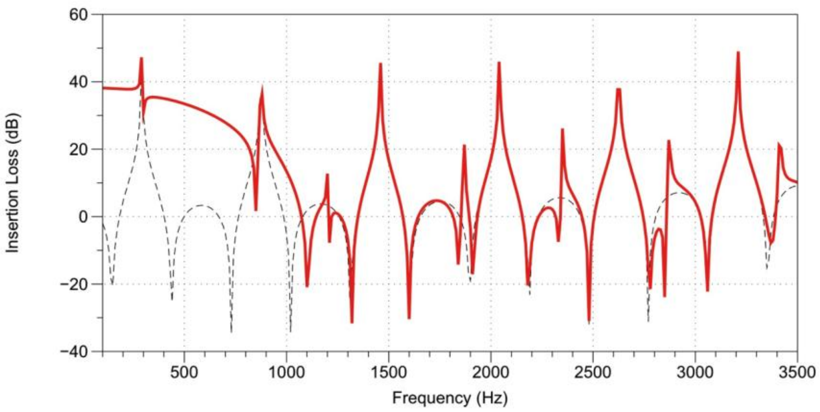

Figure3. *Computed results of IL based on (28) when the outlet locate at* $z = d/2$ *(a=0.3m, b=0.1m and d=0.6m)*

CONCLUSION

The presented here is the sound propagation in the rectangular SVU featuring an inlet and outlet is located on crossed right angle face. The outlet position was located on the larger face of the SVU in order to increase the ventilation effect and avoid the resonance of plane wave sound pressure.

The computation of insertion-loss was based on solving the wave equation considering the higher order mode effects. Proposed formular is given by equation (28) for constant velocity volume noise source. In order to further increase the sound proofing capability, the introduction of sound absorbing material and complex structures inside the SVU are required. This technology will be present in an upcoming report.

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