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# Solution of Non-Linear Unsteady Flow Equation in Surge Tank

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**ABSTRACT**

This study is concerned with the prediction of complex water hammer pressure fluctuations in surge tank. Basically such studies involve the numerical solution of nonlinear equations of continuity and momentum with the inclusion of a suitable resistance formula to evaluate energy losses due to friction. Various numerical methods of solution to study the water hammer pressure fluctuations are available. Jakobsen's numerical method appears to be better than others method. But in his solution a small term $\left(\frac{1}{4}\right)(|\Delta V|\Delta V|)$(quarter of square of velocity) has been neglected. This study presents Modified Jakobsen method, where it is shown that without neglecting this term, solution procedure can be advanced without any difficulty. Here, Modified Jakobsen's method and Jakobsen's method of solutions are developed and the results obtained for fluctuation of surge height for both the solutions are compared with existing experimental results. The close agreement with Modified Jakobsen's method clearly suggests the validity of the solution. The study also reveals that the use of constant friction factor and Nikuradse rough turbulent friction factor in the solution is not justified, whereas Barr's & Fang's explicit C-W equation gives very much close agreement. Therefore the use of Fang's (2011) resistance equation to compute friction factor is recommended.

**Keywords:** Water hammer, unsteady flow, resistance, surge tank

## INTRODUCTION

The main function of a surge tank in high head hydro power plant is to eliminate the extra water hammer pressure rise due to closer of the penstock valve to study initial flow of water to turbine. The flow condition in the pressure conduit and in the surge tank starts from turbulent transitional, when the valve at the end of the pipe is suddenly closed and finally changes from laminar to static. Therefore evaluation of the friction factor at every time step with a proper resistance equation is essential. The basic unsteady flow equations of water hammer with a surge tank considering friction are non-linear. Exact mathematical solution does not exist. The available classical solution i.e neglecting friction is impracticable since damping of surge height and velocities are not produced in classical solution. Graphical methods are very much tedious, approximate and also time consuming. Thoma's (1910) solution cannot predict the maximum surge height. Pressel (1909) used a constant value for turbulent friction factor. Jaegar's (1954) solution gives only approximate values of an upsurge and down surge. Approximate solutions given by Pearsall (1962), Sulton (1960), Prasil (1908), Warren (1915) and others and approximate charts developed by Johnson (1915), Rich et al also fail to give the correct prediction. Pickford advocated that Jacobsen's (1969) method seems to be more accurate than other numerical methods such as Escande's (1950), Pressel's (1909) and simple arithmetic mean methods. Numerical methods using computer programming provide the solutions to all these sort of problems. Das Mimi et al. (2005) also studied on the numerical solution. Therefore, in present study a numerical method namely Modified Jacobson's method is used to develop the solution of non-linear water hammer equations which is being assisted by resistance equations of Barr (1982) and Fang (2011) with the help of computer model solution (using MATLAB). The result obtained by this numerical solution is then compared with the available results of the literature and also with existing experimental data. The experimental data of Wood(1996), Martin(1983), AIT, Bangkok(1969), Borthakur and Das M M (1997) were observed, and Borthakur (1997) data was found to be suitable for numerical analysis with better experimental setup.

## GOVERNING EQUATIONS

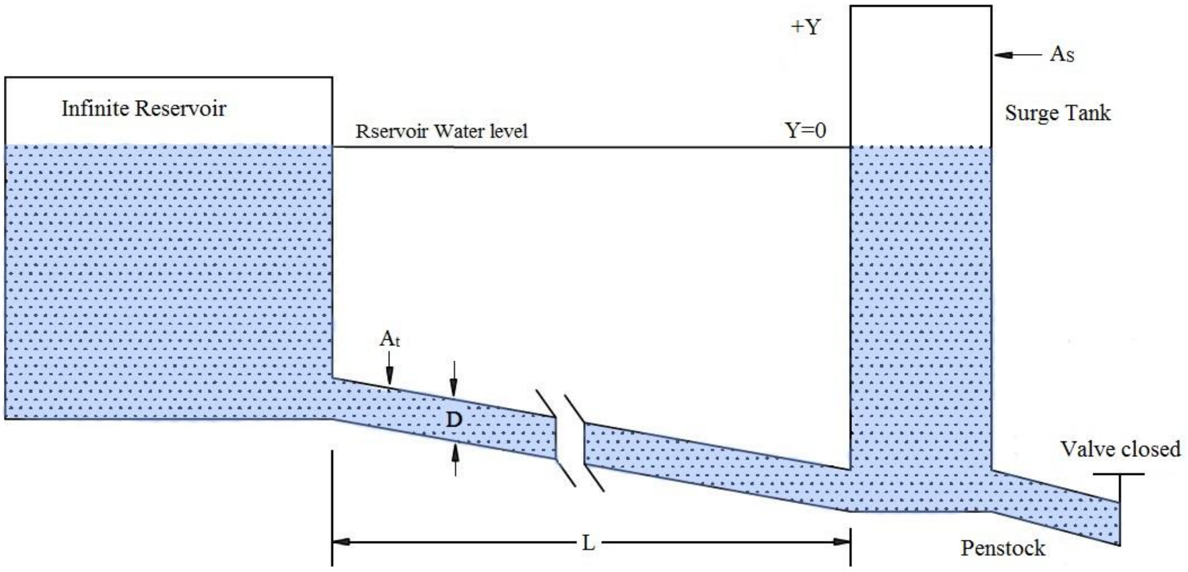


**Figure1.** *Situation before opening the penstock valve*

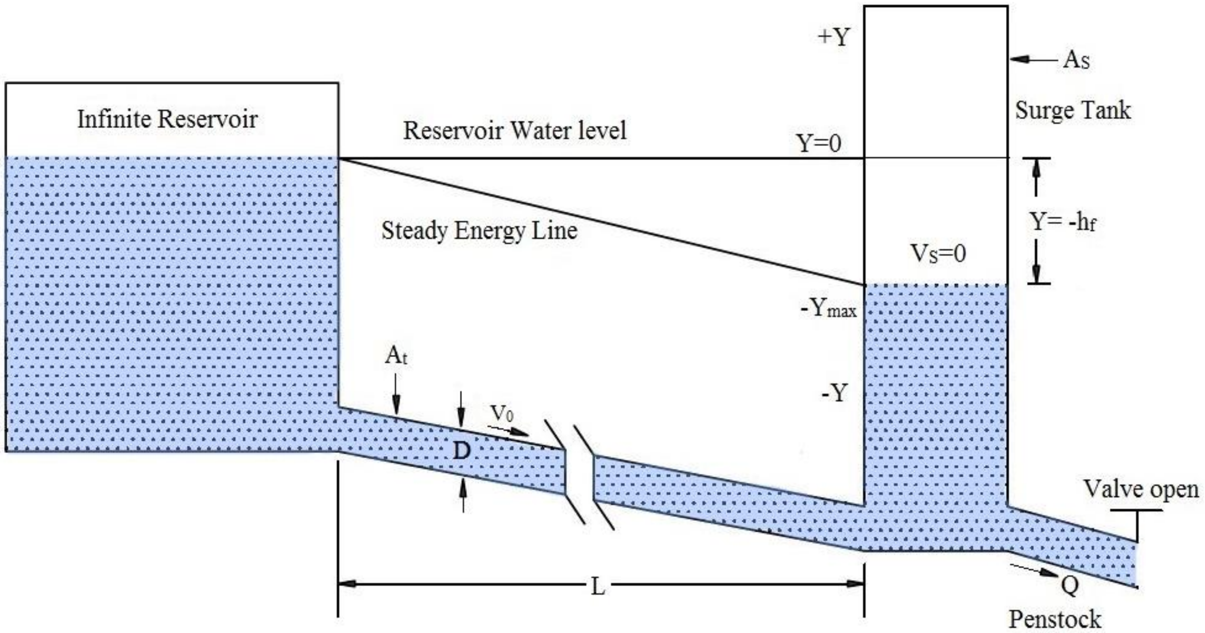


**Figure2.** *Situation of steady state when turbines are taking load uniformly*

*i.e. uniform discharge Q with steady velocity V*

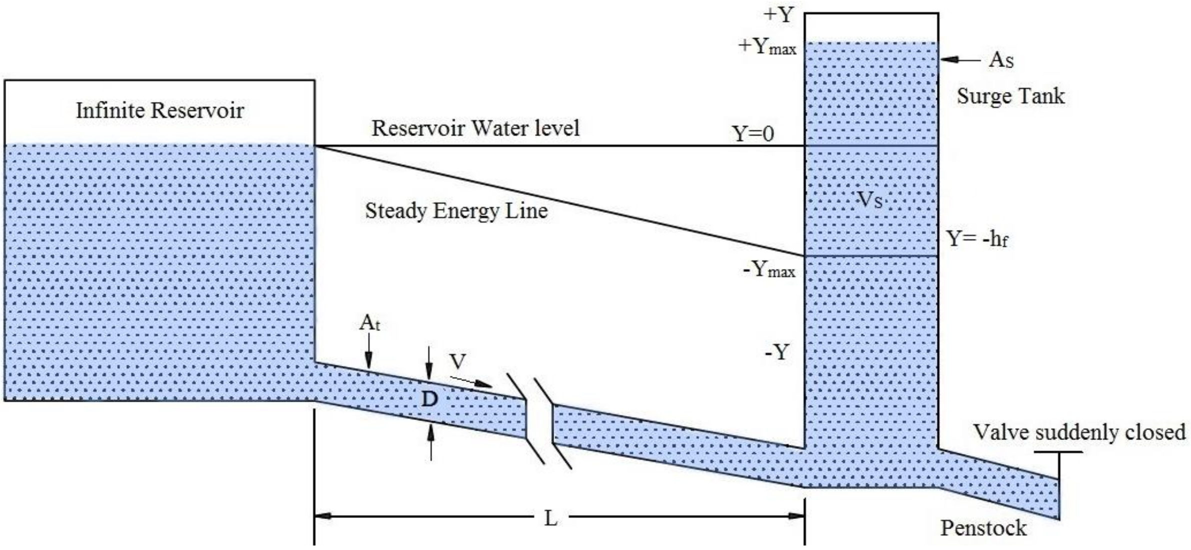


**Figure3.** *Situation of unsteady state (pressure rise in surge tank) when the valve is suddenly closed*

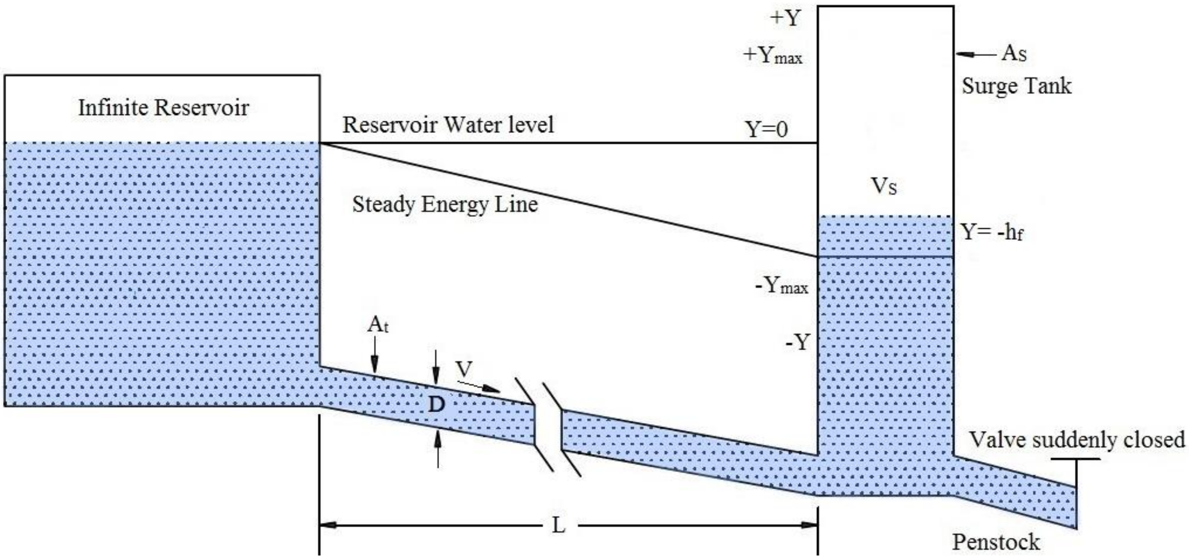


**Figure4.** *Situation of unsteady state (pressure fall in surge tank) when the valve is suddenly closed*

Figure (1) shows the situation before opening the valve. Figure (2) shows the steady state condition of flow, when the uniform discharge Q flows with the steady velocity to the power house. Figure (3) shows the unsteady flow situation at any instant after the valve is partially closed. Water enters the surge tank initially with unsteady velocity Vs and the level of water in the surge tank goes on increasing due to water hammer pressure. It goes beyond the reservoir level and surge height becomes positive. After reaching a maximum height, it begins to fall again and surge height becomes negative as shown in Fig. (4). Thus surge height within the tank moves up and down with time and ultimately due to friction it damps down to zero level. In this unsteady situation, the velocity within the tunnel or pipe changes from steady state velocity $V_o$ to unsteady velocity V. Where $A_S$ = area of the surge tank, $A_t$ = area of the pressure pipe, D= diameter of the pipe, f= friction factor, g = acceleration due to gravity, $h_f$ = head loss due to friction, L=length of the pipe line, Q = steady discharge in pressure pipe, $Q_t$ = unsteady discharge in pressure pipe, $V_0$= steady velocity in the pipe line before closing of the valve, V=unsteady velocity in the pipe at any instant after valve closure, $V_S$ = unsteady velocity in surge tank, Y= unsteady surge height at any instant after valve closure.

The basic unsteady equations of continuity and momentum for the surge tank are written respectively as:

$$v = \frac{A_s}{A_t}.\frac{dy}{dt} + \frac{Q_t}{A_t} \quad (1)$$

$$\frac{L}{g}.\frac{dV}{dt} + y + \frac{fL|V|V|}{2gD} = 0 \quad (2)$$

When the valve is completely closed, $Q_t = 0$

$$V = \frac{A_s}{A_t}.\frac{dy}{dt} \quad (3)$$

$$\therefore \left(\frac{L}{g}\right).\frac{d^2y}{dt^2}.\left(\frac{As}{At}\right) + y + \frac{fLAs^2}{2gD.At^2}.\left(\frac{dy}{dt}\right)^2 = 0$$

$$=> \frac{d^2y}{dt^2} + \frac{fAs}{2DAt}\left(\frac{dy}{dt}\right)^2 + \left(\frac{g}{L}\right).\left(\frac{At}{As}\right) \quad y = 0 \quad (4)$$

Thus combining equation (2) & (3), non-linear equation (4) is obtained and it cannot be solved analytically. Thus continuity and momentum equation becomes non-linear when friction is considered.

## MODIFIED JAKOBSEN'S METHOD

Various numerical methods of solution exist for the problem. Some of them are: **(i)** Pressel's Method of successive trials, (ii) Simple Arithmetic Method, (iii) Escande's Method, (iv) Conventional Explicit Finite Difference Method and (v) Jacobsen's Method.

According to most of the previous researchers, Jakobsen's method is simple for application and is better than other methods. But Jakobsen neglected the term $\left(\frac{1}{4}\right)(|\Delta V|\Delta V|)$ in his solutions to render the solution somewhat simpler. In present work that term is not neglected. Therefore development of this method of solution is called the "Modified Jakobsen's method".

Jakobsen expressed equations 1 and 2 in finite difference form as:

$$\bar{V} = \frac{As}{At}\frac{\Delta y}{\Delta t} \quad (5)$$

And

$$\frac{L}{g}.\frac{\Delta V}{\Delta t} + \bar{y} + \frac{fL}{2gD}\bar{V}\,|\bar{V}| = 0 \quad (6)$$

And simplifying further for $\Delta V$ and $\Delta y$

$$\Delta y = \left(V_0 + \frac{1}{2}\Delta V\right)\frac{At}{As}\Delta t \quad (7)$$

$$\Delta V = \frac{y_0 + \frac{fL}{2gD}V_0|V_0| + \frac{1}{2}V_0\Delta t^2\left(\frac{At}{As}\right)}{\frac{L}{g} + \frac{1}{4}\left(\frac{At}{As}\right)\Delta t^2 + \frac{fL}{2gD}V_0|V_0|\Delta t} \quad (8)$$

Thus knowing the initial values of $y_0$ and $V_0$ at t=0, $\Delta y$ and $\Delta V$ are calculated in next time $\Delta t$ by equation (7) & (8). Then y and V in next step $\Delta t$ are calculated from equations

$$y_n = y_{n-1} + \Delta y \tag{9}$$

$$V_n = V_{n-1} + \Delta V \tag{10}$$

Thus solution for y & V for increasing time steps may be obtained. The above techniques and equations are involved in applying the finite difference methods of Jakobsen. Thus in Modified Jakobsen's finite difference method, retaining the neglected term of Jakobsen's method, equation (8) becomes a quadratic equation in $\Delta V$, Thus

$$\Delta y = \left(V_0 + \frac{1}{2}\Delta V\right)\left(\frac{At}{As}\right)\Delta t \tag{11}$$

Solving the quadratic equation in $\Delta V$ and simplifying gives

$$\Delta V = \frac{1}{2}\left[-\left(\frac{8D}{f\Delta t} + \frac{2At\Delta tgD}{AsfL} + 4|V_0|\right)\Delta V + \sqrt{\frac{8D}{f\Delta t} + \frac{2At\Delta tgD}{AsfL} + 4|V_0|} - 4\left(\frac{8gDy}{fL} + \frac{4At\Delta tgDV_0}{AsfL} + 4|V|V_0\right)\right] \tag{12}$$

## Resistance Equations used in Proposed Method

- **Constant Friction Factor Formula**

$$\frac{1}{\sqrt{f}} = \frac{V}{\sqrt{8gD}} \tag{13}$$

- **Nikurdse's Friction Factor Formula**

$$\frac{1}{\sqrt{f_{rough}}} = 2\log_{10}\left[\frac{2.71D}{k}\right] \tag{14}$$

- **Barr's Resistance Equation**

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{k}{3.71D} + \frac{5.1286}{R^{0.89}}\right] \tag{15}$$

- **Fang's Resistance Equation**

$$f = 1.613\left[\text{In}\left(.234\left(\frac{k}{D}\right)^{1.1007} - \frac{60.525}{R^{1.1105}} + \frac{56.291}{R^{1.0712}}\right)\right]^{-2} \tag{16}$$

Where D= diameter of pipe, f = friction factor, g = acceleration due to gravity, k = average sand roughness size, R = Reynolds number.

## Assessment of Friction Factor at Low Reynolds Number

In Barr's and Fang's equations, the friction factor f is assessed for the range of Reynolds number from $2x10^3$ to $10^7$.These two equations do not consider the assessment of friction factor below the above range. When Reynolds number is less than 1500, the well known Poiseuille equation, $f = \frac{0.3164}{R^{0.25}}$ is used.

To start solution, both hydraulic and geometrical parameters of laboratory data are given as input to the computer program. Initial values are given from the steady state conditions at time t equal to zero. A very small time step (maximum 0.5 seconds) is chosen. Solutions are advanced with time up to 1000 seconds after sudden valve closure. Solutions for the variable surge height Y and surge velocity V are obtained. The solutions are demonstrated by computer plot. The values taken to start the solution are Discharge Q = 1.84675 lit/sec, head loss due to friction $h_f$ = 49 cm, area of pipe

$A_t$ = 31.669 $cm^2$, area of surge tank $A_s$ = 522.58 $cm^2$, length of pipe L = 77.25 meters, sand roughness k = 0.01483 cm, acceleration due to gravity G = 9.81m/$sec^2$, turbulent steady state friction factor f = 0.00232425, Viscosity = .000001$m^2$/Sec

## RESULTS AND DISCUSSION

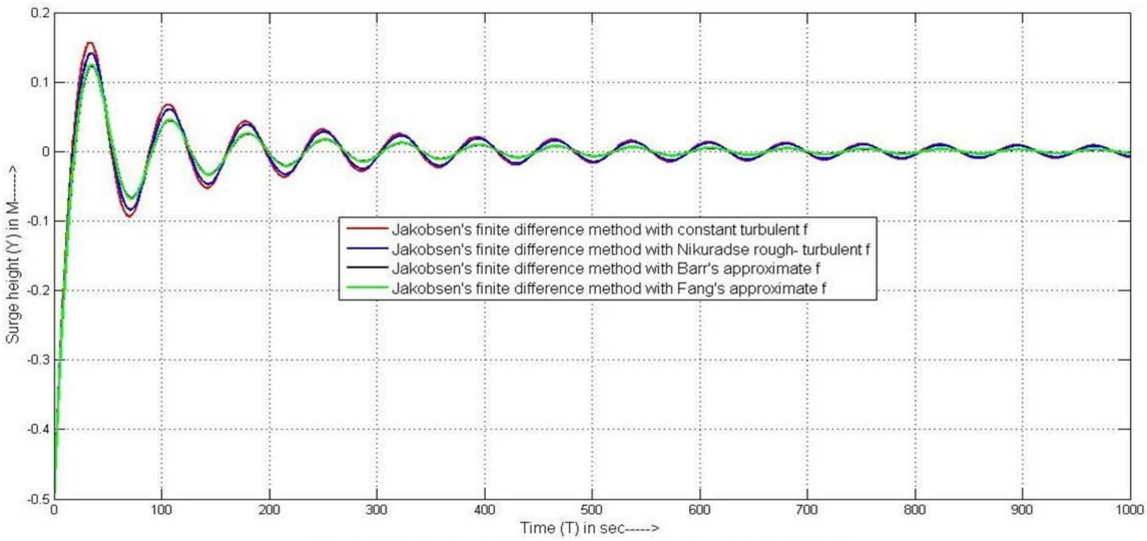


**Figure5.** *Surge height (Y) vs. time (T) by Jakobsen's method with 4 resistance equations*

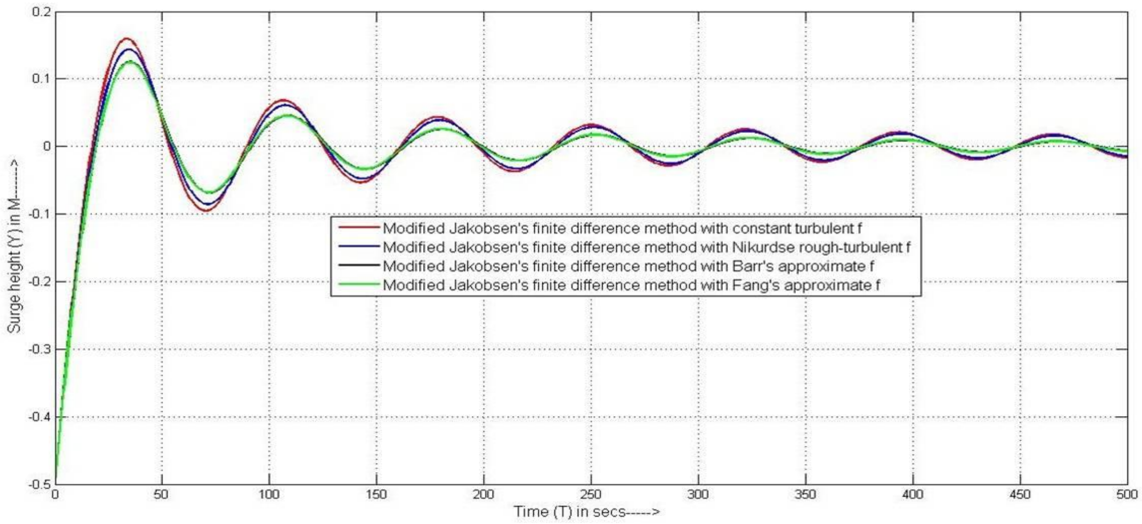


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**Figure6.** *Surge height(Y) vs. Time (T) by Modified Jakobsen's method with 4 resistance equations*

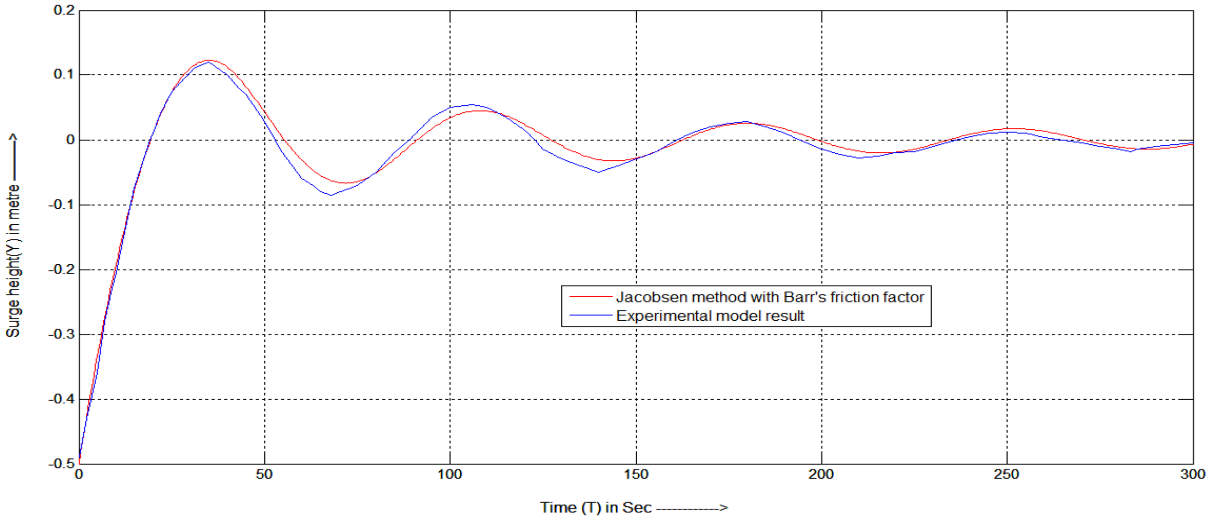


**Figure7.** *Comparison of Jakobsen's solution by Barr's resistance equation with experimental data*

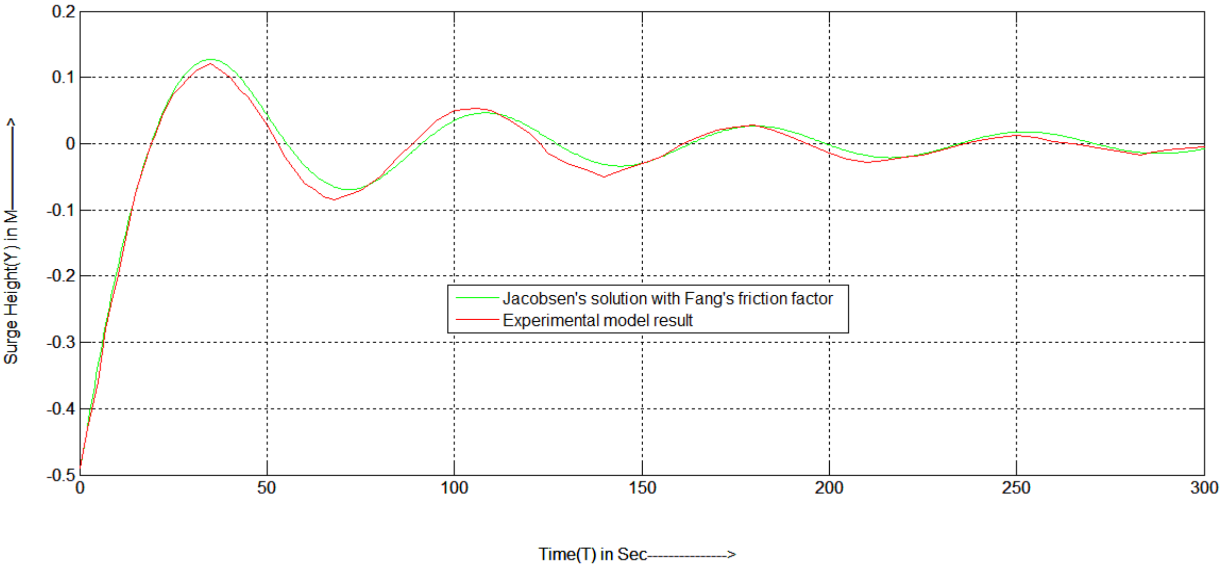


**Figure8.** *Comparison of Jakobsen's solution by Fang's resistance with experimental data*

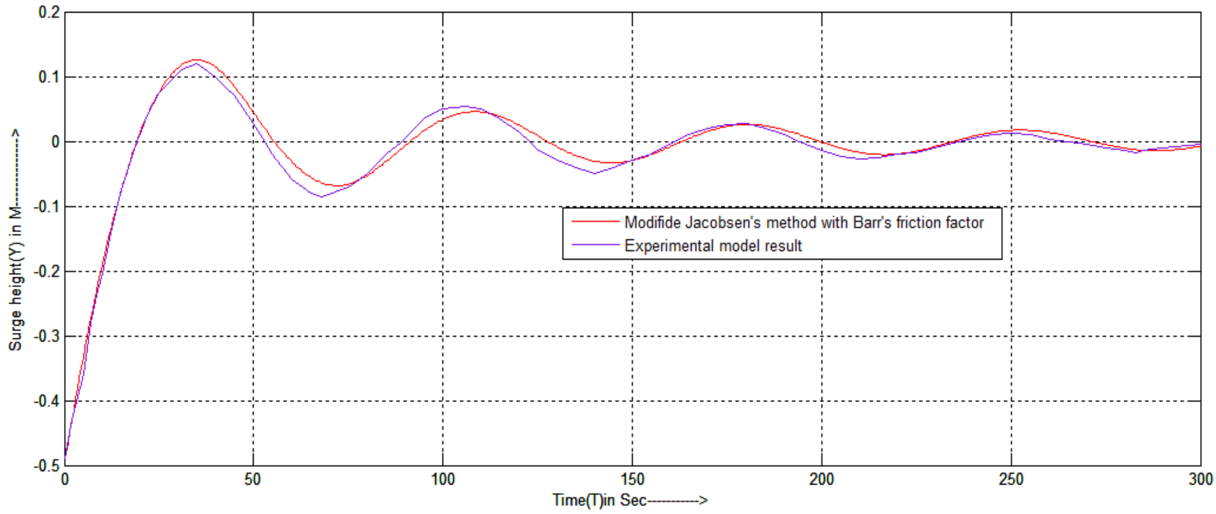


**Figure9.** *Comparison of Modified Jakobsen's solution by Barr's resistance equation with laboratory data*

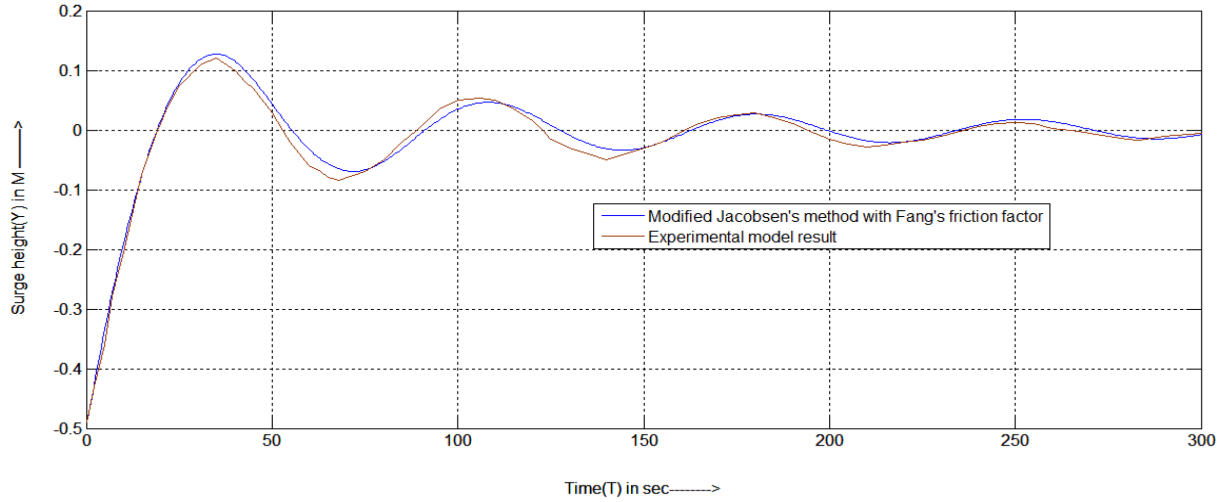


**Figure10.** *Comparison of Modified Jakobsen's solution by Fang's resistance equation with laboratory data*

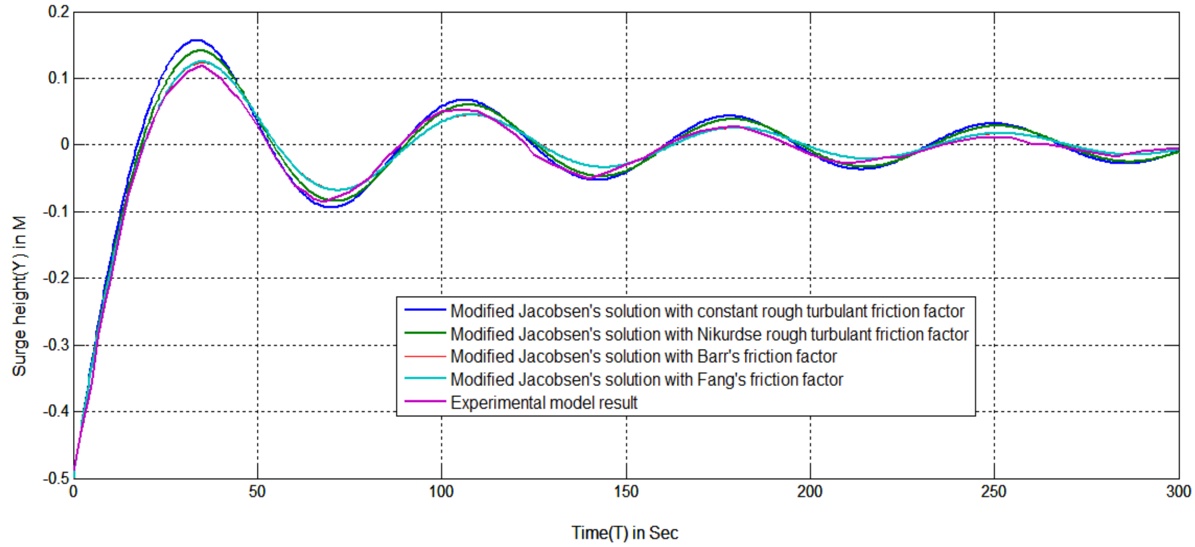


**Figure11.** *Comparison of Modified Jakobsen's solution by four resistance equations with experimental data*

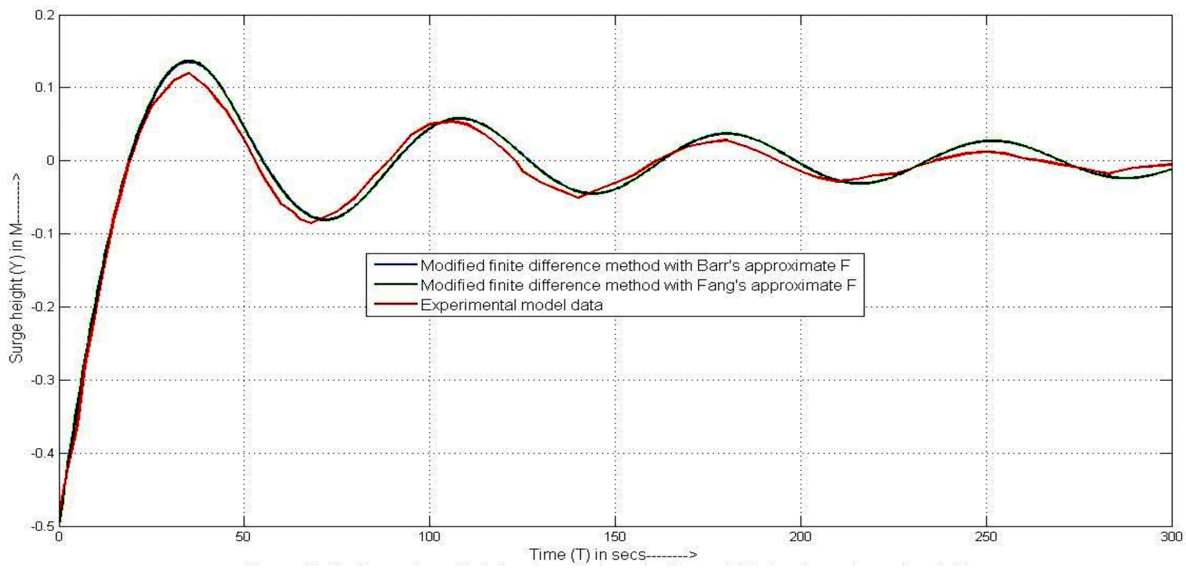


**Figure12.** *Comparison of Modified Jakobsen's solution by Barr's & Fang's resistance equations with laboratory data*

The graphical representation of the computer solution, for surge height vs. time has been represented in figure 5 and figure 6 for four friction factor used in Jakobsen's and Modified Jakobsen's method respectively. In figure 7,8,9,and 10 the results of Jakobsen's and Modified Jakobsen's solution for Barr's and Fang's explicit equations are compared with laboratory result respectively, where good compromise of numerical and experimental results are observed.From figure 11, it is seen that the maximum predicted surge height (15.7cm) is obtained with constant friction factor and also Nikuradse rough-turbulent friction factor gives the over estimation of surge height (14.1cm) than Barr's (12.3cm) and Fang's (12.4cm) friction factor in Modified Jakobsen's method and maximum surge height of experimental result is 12cm. This indicates that the use of constant friction factor and Nikuradse rough turbulent friction factor in the solution is not justified. In the solution with Barr's and Fang's resistance equation, friction factor is calculated in every time step. Finally in figure 12, the solution with Modified Jakobson's method using Barr's & Fang's explicit C-W equation is compared with experimental data which shows accurate damping of flow and gives very much close agreement. Therefore the use of Fang's (2011) resistance equation to compute friction in this work provides accurate result.

## CONCLUSION

The important point in the numerical solution in this study is the evaluation of friction factor in every time step by resistance equation. Once the friction factors are assessed at every time step, comparison is quite satisfactory, which clearly indicates the necessity for the correct assessment of friction factor at every time step. The close agreement of Modified Jakobsen's method with experimental result clearly suggest the validity of the solution.The use of Fang's (2011) resistance equation is another new approach to compute friction in this study, which provides the most accurate result.

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