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Statistical Analysis of Rainfall Data for Estimation of Peak Flood Discharge Using Rational Formula

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ABSTRACT

Estimation of Peak Flood Discharge (PFD) at a desired location on a river is important for planning, design and management of hydraulic structures. For ungauged catchments, rainfall depth becomes an important input in derivation of PFD. So, rainfall depth can be estimated through statistical analysis by fitting probability distribution to the rainfall data. In this paper, the series of annual maximum 1-day rainfall derived from daily rainfall data observed at Kasauli rain-gauge station is used for estimation of 1-day extreme rainfall adopting Gumbel distribution. Maximum likelihood method is used for determination of parameters of the distribution. Kolmogorov-Smirnov test is applied for checking the adequacy of fitting of the distribution to the observed rainfall data. The 1-day extreme rainfall obtained from Gumbel distribution is used to compute the rainfall intensity and considered as an input to estimate the PFD by rational formula adopting CWC guidelines. The study suggests the estimated PFD could be used for design of flood protection works for different tributaries of Sirsa river.

Keywords: *Kolmogorov-Smirnov test, Gumbel, Rainfall, Peak flood discharge, Maximum likelihood method*

INTRODUCTION

Estimation of design floods in ungauged catchments is frequently required in hydrological practice and is of great economic significance. These include different types of flood such as standard project flood, probable maximum flood and design basis flood. In case of large river basins, the hydrological and stream flow series of a significant duration are generally available. However, for ungauged catchments, more data is not available other than rainfall. The rainfall data is also of shorter duration and may becomes an important input in estimation of Peak Flood Discharge (PFD) [1]. For this purpose, statistical analysis involves fitting of probability distribution to the series of Annual Maximum 1-day Rainfall (AMDR) is carried out.

Out of a number of probability distributions, the family of Extreme Value Distributions (EVDs) includes Generalized Extreme Value, Extreme Value Type-1 (Gumbel), Extreme Value Type-2 (Frechet) and Extreme Value Type-3 (Weibull) is widely adopted for Extreme Value Analysis (EVA) of rainfall [2]. EVDs arise as limiting distributions for the sample of independent, identically distributed random variables, as the sample size increases. In the group of EVDs, Gumbel distribution has no shape parameter as when compared to other distributions and this means that there is no change in the shape of Probability Distribution Function (PDF). Moreover, the Gumbel distribution has the advantage of having only positive values, since the data series of rainfall are always positive (greater than zero); and therefore Gumbel distribution is important in statistics. Deka and Borah [3] have derived the best fitted distribution among the five extreme value distributions used to describe the annual series of maximum rainfall data of nine distantly located stations in north east India. Sharma and Singh [4] analyzed the series of annual, seasonal, monthly and weekly maximum rainfall data of Pantnagar region and identified the best fitted probability distribution among the sixteen distributions used in the study. Mujere [5] applied Gumbel distribution for modelling flood data for Nyanyadzi River, Zimbabwe. Baratti et al. [6] carried out flood frequency analysis at seasonal and annual time scales for Blue Nile

River adopting Gumbel distribution. Esteves [7] applied Gumbel distribution to estimate the extreme rainfall depths at different rain-gauge stations in southeast United Kingdom. Olumide et al. [8] applied normal and Log-Gumbel distributions for prediction of rainfall and runoff at Tagwai dam site in Minna, Nigeria. They have also expressed that the normal distribution as better suited for rainfall prediction while Log-Gumbel for runoff. Vivekanandan [9] applied Gumbel distribution for modelling the seasonal and annual rainfall for Krishna and Godavari river basins. Rasel and Hossain [10] applied Gumbel distribution for development of intensity duration frequency curves for seven divisions in Bangladesh. In view of the above, Gumbel distribution is used in the present study. Parameters of the Gumbel distribution are determined by Maximum Likelihood Method (MLM) and used for estimation of 1-day extreme rainfall. For quantitative assessment on rainfall data within the observed range, Kolmogorov-Smirnov (KS) test is applied. The 1-day extreme rainfall obtained from Gumbel distribution is used to compute the rainfall intensity and considered as an input to estimate the PFD for different tributaries of river Sirsa. The methodology adopted in EVA of rainfall using Gumbel distribution, assessing the adequacy of fitting of Gumbel distribution to the AMDR series using GoF test and estimation of PFD using rational formula are briefly described in the ensuing sections.

METHODOLOGY

The methodology adopted in estimation of PFD for different tributaries of river Sirsa includes (i) assess the adequacy of fitting of Gumbel distribution (using MLM) to the series of AMDR using GoF test; (ii) estimate the 1-day extreme rainfall for different return periods; (iii) compute the rainfall intensity from the 1-day extreme rainfall using CWC guidelines; (iv) estimate the PFD using rational formula; and (v) analyze the results obtained thereof.

PDF and CDF of Gumbel Distribution

The PDF and Cumulative Distribution Function (CDF) of the Gumbel distribution are given as:

$$\left.\begin{array}{l} \text{PDF}: f(r) = \dfrac{e^{-(r-\alpha)/\beta} e^{-e^{-(r-\alpha)/\beta}}}{\beta} \\ \text{CDF}: F(r) = e^{-e^{-(r-\alpha)/\beta}}, \beta > 0, \text{ where } (r = r_1, r_2, r_3, \ldots, r_N) \end{array}\right\} \quad (1)$$

where, α and β are the location and scale parameters of the distribution [11]. The parameters are computed by MLM through Equations (2) and (3), and used to estimate the rainfall (R_T) for different return periods from $R_T = \alpha + Y_T\beta$. Here, $Y_T = -\ln(-\ln(1-(1/T)))$ is called as a reduced variate for a given return period T (year).

$$\alpha = -\beta \ln\left[\sum_{i=1}^{N} \exp(-r_i/\beta) \Big/ N\right] \quad (2)$$

$$\beta = \bar{R} - \left[\sum_{i=1}^{N} r_i \exp(-r_i/\beta) \Big/ \sum_{i=1}^{N} \exp(-r_i/\beta)\right] \quad (3)$$

$$SE(R_T) = \left(\beta/\sqrt{N}\right)\left(1.15894 + 0.19187 Y_T + 1.1 Y_T^2\right)^{0.5} \quad (4)$$

where r_i is the observed AMDR of i^{th} sample, $\bar{R}$ is the average value of AMDR and N is the sample size. The lower and upper confidence limits (LCL and UCL) of the estimated extreme rainfall are obtained from the linear expressions viz.LCL=ER-1.96(SE) and UCL=ER+1.96(SE). Here, ER is the estimated Extreme Rainfall and SE the Standard Error.

Goodness-of-Fit Test

GoF test is essential for checking the adequacy of probability distribution to the observed series of AMDR. Out of a number GoF tests available, the widely accepted GoF test is KS, which is used in the study. The theoretical description of KS test statistic [12] is as follows:

$$KS = \underset{i=1}{\overset{N}{\text{Max}}}\left(F_e(r_i) - F_D(r_i)\right) \quad (5)$$

where, $F_e(r_i) = i/(N+1)$ is the empirical CDF of r_i and $F_D(r_i)$ is the computed CDF of r_i. If the computed value of KS test statistic given by Gumbel distribution is less than that of the theoretical value at the desired significance level then the distribution is considered to be acceptable for modelling the series of AMDR.

Application

In this paper, a study on estimation of PFD for different return periods for different tributaries of Sirsa was carried out. The series of Annual AMDR was extracted from the daily rainfall data observed at Kasauli rain-gauge station during the period 1991 to 2015 and used. The descriptive statistics such as average, standard deviation, coefficient of variation, coefficient of skewness and coefficient of kurtosis of the observed AMDR was determined as 93.8 mm, 52.1 mm, 55.5%, 1.523 and 4.472 respectively. From the scrutiny of rainfall data, it was observed that the rainfall data for the years 2004 and 2005 are not available. Therefore, the data for the missing year was replaced with the maximum value of AMDR series, i.e, 262 mm.

The AMDR series with imputed value was used for estimation of 1-day extreme rainfall. Figure 1 presents the time series plot of the observed AMDR.

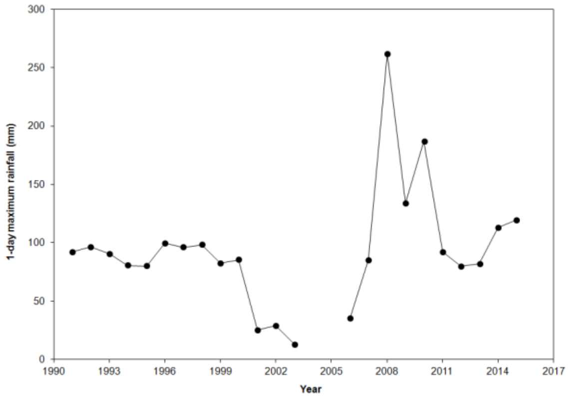

Figure1. *Time series plot of observed AMDR*

The estimated 1-day extreme rainfall obtained from Gumbel distribution (using MLM) was used as an input to estimate the PFD. The study involves flood protection works for Sirsa river for which the survey data of the river was collected and used. Sirsa is a tributary to river Satluj. There are 12 tributaries to Sirsa that contribute to flood flows. For estimating the floods, catchment characteristics (catchment area, length of stream, slope, etc.) area required, which was extracted from the Survey of India (SoI) toposheets of the region. The catchment areas of the tributaries of Sirsa river were given in Table 1.

Table1. *Catchment area of tributaries of Sirsa river*

Tributary ID	Name of the tributary	Catchment Area (km^2)
1	Sirsa river at RD 28300	241.00
1 (a)	Sirsa river at RD 18000	290.15
1 (b)	Sirsa rier at RD 10000	354.32
1 (c)	Sirsa rier at RD 5000	464.62
1 (d)	Sirsa river at RD 0	521.11
2	Chikni Khad at RD 500	94.74
3	Khera Khad at RD 500	15.56
4	Bagbania Khad at RD 500	23.37
5	Manpura Khad at RD 500	22.53
6	Ratta main river at RD 500	22.61
7	Sandholi Khad at RD 500	7.40
8	Pali Mahadev at RD 500	56.49

RESULTS AND DISCUSSIONS

By applying the procedures of Gumbel distribution, parameters were determined by MLM and used for estimation of 1-day extreme rainfall.

Estimation of 1-day Extreme Rainfall Using Gumbel Distribution

Table 2 gives the 1-day extreme rainfall estimates with confidence limits for different return periods vary from 2-year to 100-year adopting Gumbel distribution. The observed and estimated AMDR are presented in Figure 2 along with confidence limits. From Figure 2, it can be seen that about 75 percent of the observed AMDR are within the confidence limits of the estimated 1-day extreme rainfall using Gumbel distribution.

Table2. *Estimated 1-day extreme rainfall (mm) with confidence limits using Gumbel distribution*

Return period (yr)	Estimated Rainfall (ER)	Standard Error on ER	Confidence limits at 95% level	
			Lower	Upper
2	86.2	9.9	66.9	105.5
5	131.8	15.1	102.2	161.4
10	162.0	19.4	124.0	200.0
20	191.0	23.7	144.5	237.5
25	200.2	25.1	150.9	249.4
50	228.5	29.5	170.6	286.3
75	244.9	32.1	182.0	307.8
100	256.6	33.9	190.1	323.0

Analysis Based on GoF Test

The adequacy of fitting of Gumbel distribution to the series of AMDR was performed by adopting KS test, as described above. From the result, it was observed that the computed value of KS test statistic is 0.199, which is not greater than the theoretical value of 0.264 at 5% significance level [13], and at this level, the KS test confirmed the suitability of Gumbel distribution for modelling the series of AMDR.

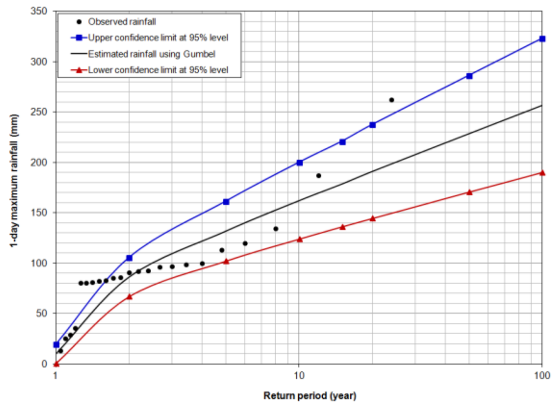

Figure2. *Plots of observed and estimated 1-day extreme rainfall with confidence limits*

Estimation of Peak Flood Discharge

The requirement of the study is to estimate the PFD for Sirsa and its tributaries. Neither Sirsa nor its tributaries are gauged and thus needs to be estimated by indirect estimations from rainfall and catchment characteristics. From an observation of catchment size from the Toposheets and Google Earth of the region of these tributaries it was found that these are small catchments that respond quickly to rainfall. From the catchment characteristics, the time lag (t_L) and time of concentration (t_c) were estimated for each of these catchments. As the present study is concerned with peak flood estimation the design storm durations for each of the catchment were estimated from t_L. From the analysis of catchment characteristics, the computed design storm durations for different catchments vary from 1-hour (hr) to 6-hr. The design storm duration is 1-hr for Bagbania Khad, Manpura Khad, Ratta main river, Sandholi Khad and Pali Mahadev Khad; 2-hr for Khera Khad and Sirsa RD 28300; 3-hr for Chikni Khad; 4-hr for Sirsa RD 18000; 5-hr for Sirsa RD 10000; and 6-hr for Sirsa RD 5000 and Sirsa RD 0. It is to state that, the shorter duration rainfall (say, 1-hr) were not available for the study area. Based on the design storm durations, the appropriate storms (2-hr, 3-hr, 4-hr, 5-hr and 6-hr) were estimated adopting the procedures as detailed in CWC report [14] and used for computing the design storm of the respective catchments from the estimated 1-day extreme rainfall (100-year return period) by using suitable conversion factors (Figure 3), as given in Central Water Commission Report titled ‘Flood estimation report for Western Himalayas Zone - 7’. The computed values of distributed rainfall are presented in Table 3. This value was used as input (rainfall intensity) for estimation of PFD for the catchments of Sirsa river and its tributaries. The catchment areas are in the range of 7.40 km^2 to 521.11 km^2, the largest of the catchment being Sirsa river at RD 0 and the smallest catchment being Sandholi Khad at RD 500. As mentioned these tributaries are ungauged and hence the PFD for these catchments are computed by using rational formula, which is given below:

$$q = 0.278 * C\,I\,A \qquad (6)$$

where, q is PFD (m^3/s), C is runoff coefficient, I is rainfall intensity (mm/hour) and A is catchment area (km^2).

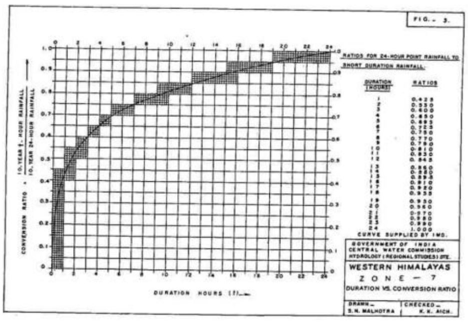

Figure3. *Conversion factor for computation of distributed rainfall for shorter duration*

Table3. *Distributed rainfall for short durations*

Return period (year)	Distributed rainfall (mm) for short durations based on design storm					
	1-hr	2-hr	3-hr	4-hr	5-hr	6-hr
2	36.6	39.8	37.8	34.7	33.5	23.7
5	56.0	60.9	57.7	53.1	51.3	36.3
10	68.8	74.8	71.0	65.3	63.1	44.6
15	75.9	81.7	77.4	71.3	68.8	48.7
20	81.2	88.2	83.7	77.0	74.3	52.6
25	85.1	92.5	87.7	80.7	77.9	55.2
50	97.1	105.6	100.1	92.1	88.9	63.0
75	104.1	113.1	107.3	98.7	95.3	67.5
100	109.0	118.5	112.4	103.4	99.9	70.7

By considering topography and general land use of the catchments, the value of C is considered as 0.55 for estimation of flood discharge. The computed PFD for different return periods for tributaries of Sirsa river are presented in Table 4. It is to state that the design discharge [15] may be appropriately chosen with a particular return period. EVA of rainfall and hydrologic consideration adopting the design storms estimated for the catchments with appropriate return period may be used for flood protection works.

Table4. *Peak flood discharge for different return periods for tributaries of Sirsa river*

Name of the Tributary	Peak flood discharge (10^3 m^3/s) for different return periods							
	2	5	10	20	25	50	75	100
Sirsa river at RD 28300	1.467	2.244	2.758	3.252	3.408	3.890	4.169	4.368
Sirsa river at RD 18000	1.541	2.356	2.896	3.415	3.579	4.085	4.378	4.588
Sirsa rier at RD 10000	1.623	2.481	3.050	3.596	3.769	4.302	4.610	4.831
Sirsa river at RD 5000	1.687	2.580	3.171	3.738	3.918	4.472	4.793	5.022
Sirsa river at RD 0	1.892	2.893	3.556	4.193	4.395	5.016	5.376	5.633
Chikni Khad at RD 500	0.547	0.836	1.028	1.212	1.270	1.450	1.554	1.628
Khera Khad at RD 500	0.095	0.145	0.178	0.210	0.220	0.251	0.269	0.282
Bagbania Khad at RD 500	0.131	0.200	0.246	0.290	0.304	0.347	0.372	0.390
Manpura Khad at RD 500	0.126	0.193	0.237	0.280	0.293	0.335	0.359	0.376
Ratta Main River at RD 500	0.127	0.194	0.238	0.281	0.294	0.336	0.360	0.377
Sandholi Khad at RD 500	0.041	0.063	0.078	0.092	0.096	0.110	0.118	0.123
Pali Mahadev at Rd 500	0.316	0.484	0.595	0.701	0.735	0.839	0.899	0.942

CONCLUSIONS

The paper described briefly the study carried out for statistical analysis of rainfall data adopting Gumbel distribution and estimation of PFD for tributaries of river Sirsa. From the results of the data analysis, the following conclusions were drawn from the study:

i) The KS test result confirmed the suitability of Gumbel distribution (using MLM) for modelling the data series of AMDR.

ii) From Figure 2, it was observed that about 75 percent of the observed AMDR data are within the confidence limits of the estimated 1-day extreme rainfall.

iii) The 1-day extreme rainfall was used to compute the values of the distributed rainfall adopting CWC guidelines described in the 'Flood estimation report for Western Himalayas-Zone 7'.

iv) By using the rational formula, the PFD for different return periods for different tributaries of Sirsa river was estimated.

v) The study suggested that the PFD, as given in Table 4, could be considered for design of flood protection works. However, by considering the data length made available for the study, it was cautioned to use the PFD for return periods beyond 50-year because of uncertainty in the estimated values.

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