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Comparison of MLM Estimators of Probability Distributions for Assessment of Extreme Rainfall

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ABSTRACT

Assessment of Extreme Rainfall (ER) for a desired return period is of utmost importance for planning and design of minor and major hydraulic structures such as dams, bridges, barrages, storm water drainage systems. Depending on the design-life of structure, ER with a desired return period is used. This can be achieved through Extreme Value Analysis (EVA) of rainfall by fitting of probability distributions to the observed annual 1-day maximum rainfall series. In this paper, efforts have been made to compare the Maximum Likelihood Method (MLM) estimators of Gumbel (EV1), Frechet, 2-parameter Log Normal (LN2) and Log Pearson Type-3 (LP3) probability distributions for assessment of ER at Fatehabad, Hansi, Hissar and Tohana sites. The adequacy of probability distribution used for EVA of rainfall is assessed by quantitative assessment using Goodness-of-Fit tests viz., Anderson-Darling and Kolmogorov-Smirnov, and diagnostic test using D-index. Based on quantitative assessment, the study suggested that the LN2 is better suited probability distribution for assessment of ER at Fatehabad whereas EV1 for Hansi and LP3 for Hissar and Tohana.

Keywords: *Anderson-Darling, D-index, Extreme Value Analysis, Gumbel, Kolmogorov-Smirnov, Log-Normal, Log-Pearson Type-3, Rainfall*

INTRODUCTION

Assessment of Extreme Rainfall (ER) for a desired return period is a pre-requisite for planning, design, management and operation of various hydraulic structures such as dams, bridges, barrages, storm water drainage systems, etc. Depending on the size and design-life of structure, the estimated ER corresponding to a desired return period (T) is used. Generally, 1000-year return period estimated ER will be considered for the design of hydraulic structures having a design life of 1000-year. The estimation of requisite return period rainfall depth could be achieved through Extreme Value Analysis (EVA) by fitting probability distributions to the observed Annual 1-Day Maximum Rainfall (ADMR).

Out of a number of probability distributions, Gumbel (EV1), Frechet (EV2), Log Normal (LN2) and Log Pearson Type-3 (LP3) distributions are generally applied for EVA of rainfall [1-2]. Number of studies has been carried out by different researchers which showed that there is no unique distribution available for EVA of rainfall for a region or country [3-4]. Lee [5] indicated that LP3 distribution fits for 50% of total station number for the rainfall distribution characteristics of Chia-Nan plain area. Bhakar et al. [6] studied the frequency analysis of consecutive day's maximum rainfall at Banswara, Rajasthan. Study by Saf et al. [7] revealed that the Pearson Type-3 distribution is better suited for modelling of extreme values in Antalya and Lower-West Mediterranean sub-regions whereas the Generalized Logistic distribution for the Upper-West Mediterranean sub-region. Varathan et al. [8] found that the EV1 distribution is the best fitting distribution to analyse the annual maximum rainfall of Colombo district. AlHassoun [9] carried out a study on developing empirical formulae to estimate rainfall intensity in Riyadh region using EV1, LN2 and LP3. He concluded that the LP3 distribution gives better accuracy amongst three distributions studied in estimation of rainfall intensity. Baratti et al. [10] carried out flood frequency analysis on seasonal and annual time scales for the Blue Nile River adopting EV1 distribution. Esteves [11] applied EV1 distribution to estimate the ER depths at different rain-gauge stations in southeast United Kingdom. Rasel and Hossain [12] applied EV1 distribution for development of intensity duration frequency curves for seven divisions in Bangladesh. Moreover, when number of parameter estimation procedures of different probability distributions used in EVA of rainfall, a common problem that arises is how to determine which model fits best for a given set of data. This possibly could be answered by Goodness-of-Fit (GoF) tests and the results are quantifiable and reliable. GoF tests viz., Anderson-Darling (A^2) and Kolmogorov-Smirnov (KS) are applied for checking the adequacy of fitting of probability distributions to the observed ADMR [13]. In addition to A^2 and KS tests, a diagnostic test of D-index is used for identifying the best suitable probability distribution for EVA of rainfall. The literature thus suggests a varied spectrum of applications of Probability Distribution Functions (PDFs) and the parameter estimation methods to achieve the desired goals of extreme event of the variable under consideration. In this paper, the procedures adopted in the probabilistic analysis of ER using GoF tests are demonstrated with illustrative example.

METHODOLOGY

In this paper, a study has been carried out to assess the PDFs adopted in EVA of rainfall. For this, it is required to process and validate the data of the variate for application such as (i) perform statistical tests such as independency, homogeneity and outliers; (ii) determine the parameters of EV1, EV2, LN2 and LP3 distributions by Maximum Likelihood Method (MLM); (iii) Evaluate the EVA results obtained from PDFs through GoF and diagnostic tests, and recommendations made thereof. Table 1 gives the PDFs with the corresponding quantile estimators (R_T) of PDFs used in the EVA study.

Table1. *PDF and quantile estimator used for EVA*

Distribution	PDF	Quantile estimator
EV1	$f(r;\alpha,\beta)=\frac{e^{-(r-\alpha)/\beta}e^{-e^{-(r-\alpha)/\beta}}}{\beta}$, β >0	$R_T=\alpha+Y_T\beta$
EV2	$f(r;\beta,\gamma)=\frac{\gamma}{\beta}\left(\frac{\beta}{r}\right)^{\gamma+1}e^{-\left(\frac{r}{\beta}\right)^{-\gamma}}$, β >0	$R_T=\beta e^{(Y_T/\gamma)}$
LN2	$f(r;\alpha,\beta)=\frac{1}{\beta\sqrt{2\pi}}\exp\left(-\frac{(r-\alpha)^2}{2\beta^2}\right)$,-∞<r<∞, β >0	$R_T=e^{\alpha+K_P\beta}$
LP3	$f(r;\alpha,\beta,\gamma)=\frac{1}{\beta r\Gamma\gamma}\left(\frac{\ln(r)-\alpha}{\beta}\right)^{\gamma-1}e^{-\left(\frac{\ln(r)-\alpha}{\beta}\right)}$, β, γ, >0	$R_T=e^{(\alpha+\beta\gamma)+K_P\beta\sqrt{\alpha}}$

In Table 1, the symbols viz.,α, β and γ denote the location, scale and shape parameters of the distributions respectively. For EV1 and EV2 distributions, the reduced variate (Y_T) for a given return period (T) is defined by Y_T =-ln(-ln(1-(1/T))) while in the mathematical representation of LN2 and LP3, K_P denotes the frequency factor corresponding to the probability of exceedance. The Coefficient of Skewness (C_S) is C_S=0.0 for LN2 whereas C_S is based on the log transformed series of the observed data for LP3 [14].

Goodness-of-Fit Tests

Generally, A^2 test is applied for checking the adequacy of fitting of EV1 and EV2 distributions. The procedures involved in application of A^2 test for LN2 and LP3 are more complex though the utility of the test is extended for checking the quantitative assessment. In view of the above, KS test is widely applied for the purpose of quantitative assessment. Theoretical descriptions of GoF tests are as follows:

A^2 test statistic is defined as below:

$$A^2=(-N)-(1/N)\sum_{i=1}^{N}\begin{Bmatrix}(2i-1)\ln(Z_i)+\\(2N+1-2i)\ln(1-Z_i)\end{Bmatrix} \quad \ldots (1)$$

Here, $Z_i=F(r_i)$ for i=1,2,3,…,N with $r_1<r_2<\ldots<r_N$, $F(r_i)$ is the Cumulative Distribution Function (CDF) of i^{th} sample (r_i) and N is the sample size.

KS test statistic is defined as below:

$$KS=\underset{i=1}{\overset{N}{Max}}\,(F_e(r_i)-F_D(r_i)) \quad \ldots (2)$$

Here, $F_e(r_i)$ is the empirical CDF of r_i and $F_D(r_i)$ is the derived CDF of r_i by PDFs. In this study, Weibull plotting position formula is used for computation of empirical CDF.

Test criteria: If the computed value of GoF test statistic given by the distribution is less than that of theoretical value at the desired significance level then the distribution is assumed to be suitable for EVA of rainfall at that level of significance.

Diagnostic Test

Sometimes the GoF test results would not offer a conclusive inference thereby posing a bottleneck issues to the user in selecting the suitable PDF for application. In such cases, a diagnostic test in adoption to GoF is applied for making inference. The selection of most suitable probability distribution for EVA of rainfall is performed through D-index test [15], which is given as below:

$$\text{D-index} = \left(1/\overline{R}\right)\sum_{i=1}^{6}\left|r_i - r_i^*\right| \qquad \ldots (3)$$

Here, $\overline{R}$ is the average value of the observed ADMR whereas r_i (i= 1 to 6) and r_i^* are the six highest observed and corresponding estimated ER by different PDFs. The distribution having the least D-index is considered as better suited distribution for EVA of rainfall.

APPLICATION

EVA of rainfall data was carried out to estimate the ER (R_T) adopting EV1, EV2, LN2 and LP3 PDFs. MLM was used for determination of parameters of the distributions. Daily rainfall data (with missing values) observed at Fatehabad and Hansi for the period 1954 to 2011, Hissar for the period 1969 to 2011 and Tohana for the period 1951 to 2011 was used. The ADMR series was extracted from the daily rainfall data and used for EVA. From the scrutiny of the daily rainfall data, it was observed that the data for the intermittent period for Fatehabad and Hansi (1966 and 1967), Hissar (2002) and Tohana (1958 to 60 and 1966 to 67) are missing. So, the data for the missing years were imputed by the series maximum value of 140 mm (for Fatehabad), 228.6 (for Hansi), 256.5 mm (for Hissar) and 158.8 mm (for Tohana). Thereafter, the data series with imputation was used for EVA. Table 2 gives the descriptive statistics of ADMR.

Table2. *Descriptive Statistics of ADMR*

Site	Average (mm)	SD (mm)	CV (%)	C_S	C_K
Fatehabad	62.0	31.2	50.2	0.850	0.473
Hansi	62.4	51.0	81.7	2.225	4.868
Hissar	93.8	56.4	60.1	1.631	2.320
Tohana	73.2	39.6	54.1	0.932	0.119

SD: Standard Deviation; CV: Coefficient of Variation; C_S: Coefficient of Skewness; C_K: Coefficient of Kurtosis.

Data Validation

The data series used for EVA should satisfy certain basic assumption such as data should be independent and identically distributed with the meteorological process (rainfall). The term independent denotes that no observation in the data series has any influence on any other observation following i.e., the data series are random. Similarly, homogeneity of the sample elements in the data series has to be checked to identify whether the data originates from a single population or not. The presence of outliers in a data sample has undesirable effect on frequency analysis. Therefore, the sample also needs to be checked for outliers if any. In the present study, Wald-Wolfowitz and Mann-Whitney Wilcoxon tests were used for checking the randomness and homogeneity of the data series of ADMR. Grubbs test was used for detection of outliers in the data series [16]. Table 3 gives the statistical test results for the series of ADMR. From Table 3, it is observed that the computed values of Wald-Wolfowitz and Mann-Whitney Wilcoxon tests for the series of ADMR (Tohana) are lesser than the theoretical value (either 5 % or 1 % level); and at this level, the data series were found to be random and homogeneous. Further, the test results indicated that the series of ADMR for Fatehabad and Hissar was not random but homogeneous. Similarly, the series of ADMR of Hansi was found to be random but not homogenous. The Grubbs test results showed that there are some outliers in the rainfall series of Hansi and Hissar. However, the entire data was used for EVA considering the importance of the actually observed extremes in the region under consideration.

RESULTS AND DISCUSSIONS

The procedures described above for estimating ER have been implemented adopting computer codes and used in EVA of rainfall. The program computes the (i) statistical tests results for the data series; (ii) parameters of EV1, EV2, LN2 and LP3 distributions; (iii) ER estimates with Standard Error (SE) for different return periods; and (iv) GoF and D-index tests values.

Estimation of ER

The analysis of ADMR series passed the statistical tests required for EVA. The parameters of EV1, EV2, LN2 and LP3 distributions were determined by MLM that are used for estimation of ER and the EVA results are presented in Tables 4 to 7.

Analysis Based on GoF Tests

The GoF tests statistic values of EV1, EV2, LN2 and LP3 distributions were computed by using Eqs. (1) and (2), and the results are presented in Table 8. Based on GoF tests results, it may be noted that:

i) The A^2 test confirmed the use of LN2 and LP3 distributions for EVA of rainfall for all four sites considered in the study.

ii) The A^2 test results also indicated the EV2 distribution is not found to be acceptable for EVA of rainfall for Fatehabad, Hissar and Tohana.

iii) The KS test suggested the EV1, EV2, LN2 and LP3 distributions are acceptable for EVA of rainfall for all four sites considered in the study.

Table3. *Statistical test results for randomness, homogeneity and outliers*

Data series	Wald-Wolfowitz				Mann-Whitney Wilcoxon				Grubbs test (Detection of outliers)
	Com-puted value	Theo-retical value	Signifi-cance level	Random-ness of data	Com-puted value	Theo-retical value	Signi-ficance level	Homo-geneity of data	
Fatehabad	2.495	2.330	1 %	No	1.409	1.960	5 %	Yes	No outliers
Hansi	2.090	2.330	1 %	Yes	2.769	2.330	1 %	No	Yes(228.6)
Hissar	2.734	2.330	1 %	No	1.308	1.960	5 %	Yes	Yes(256.5)
Tohana	2.225	2.330	1 %	Yes	1.450	1.960	5 %	Yes	No outliers

Table4. *Estimated ER (mm) with SE (mm) by EV1, EV2, LN2 and LP3 distributions for Fatehabad*

Return period (year)	EV1		EV2		LN2		LP3	
	ER	SE	ER	SE	ER	SE	ER	SE
2	57.0	3.7	49.7	3.4	54.3	3.9	56.9	5.5
5	84.2	6.3	80.2	9.1	85.8	7.1	86.2	8.8
10	102.3	8.5	110.2	16.7	108.9	10.5	104.5	12.3
20	119.6	10.7	149.5	28.6	132.6	14.6	121.0	16.2
50	142.1	13.6	221.7	55.0	165.6	20.9	140.9	21.4
100	158.9	15.9	298.0	87.2	192.0	26.5	154.8	25.3
200	175.6	18.2	400.0	135.5	219.9	32.7	167.9	29.2
500	197.7	21.2	589.8	237.7	259.1	42.1	184.1	34.2
1000	214.4	23.5	791.0	359.2	290.7	50.1	195.6	38.0

Table5. *Estimated ER (mm) with SE (mm) by EV1, EV2, LN2 and LP3 distributions for Hansi*

Return period (year)	EV1		EV2		LN2		LP3	
	ER	SE	ER	SE	ER	SE	ER	SE
2	54.1	6.1	45.4	3.7	49.3	4.3	47.4	4.8
5	98.8	10.3	90.1	12.1	86.0	8.7	79.6	10.3
10	128.4	13.9	141.8	25.6	115.1	13.5	106.8	16.4
20	156.8	17.5	219.1	50.4	146.4	19.5	137.8	24.4
50	193.5	22.4	384.9	115.1	191.8	29.5	186.0	38.6
100	221.1	26.0	587.1	207.9	229.8	38.6	228.9	52.6
200	248.5	29.7	894.2	368.1	271.0	49.2	278.5	69.8
500	284.7	34.6	1557.6	766.3	331.0	65.7	355.7	98.7
1000	312.0	38.4	2369.3	1317.1	380.9	80.1	424.3	126.2

Table6. *Estimated ER (mm) with SE (mm) by EV1, EV2, LN2 and LP3 distributions for Hissar*

Return period (year)	EV1		EV2		LN2		LP3	
	ER	SE	ER	SE	ER	SE	ER	SE
2	84.0	6.4	77.6	7.1	81.0	6.6	80.2	8.7
5	124.7	10.8	139.0	21.0	126.8	12.0	127.4	17.0
10	151.6	14.6	204.6	41.6	160.2	17.6	162.8	25.7
20	177.4	18.5	296.3	76.8	194.4	24.2	199.9	36.0
50	210.9	23.6	478.5	161.9	241.6	34.6	252.3	52.6
100	235.9	27.5	685.3	275.3	279.3	43.7	295.0	67.6
200	260.9	31.4	980.2	459.1	318.9	54.0	340.8	84.7
500	293.8	36.6	1571.7	883.2	374.5	69.3	406.3	111.2
1000	318.7	40.6	2245.7	1430.1	419.2	82.3	460.1	134.0

Table7. *Estimated ER (mm) with SE (mm) by EV1, EV2, LN2 and LP3 distributions for Tohana*

Return period (year)	EV1		EV2		LN2		LP3	
	ER	SE	ER	SE	ER	SE	ER	SE
2	66.8	4.6	60.3	5.0	63.4	4.5	65.3	6.0
5	101.5	7.8	112.2	15.1	100.5	8.2	101.3	10.6
10	124.5	10.5	169.3	30.6	127.8	12.1	125.6	15.1
20	146.6	13.2	251.1	57.7	155.9	16.7	148.9	20.3
50	175.1	17.0	418.5	124.7	194.9	24.1	178.9	27.8
100	196.5	19.8	613.5	216.5	226.2	30.6	201.3	33.9
200	217.9	22.5	898.3	368.3	259.2	37.9	223.5	40.5
500	246.0	26.3	1485.4	727.6	305.8	48.7	252.7	49.6
1000	267.2	29.1	2172.3	1202.1	343.4	58.0	274.7	56.8

Table8. *GoF tests statistic values given by EV1, EV2, LN2 and LP3 distributions*

Site	GoF tests statistic values given by							
	EV1		EV2		LN2		LP3	
	A^2	KS	A^2	KS	A^2	KS	A^2	KS
Fatehabad	0.208	0.036	2.700	0.144	0.416	0.082	0.228	0.058
Hansi	2.446	0.135	0.532	0.089	0.419	0.082	0.367	0.087
Hissar	0.868	0.079	0.900	0.104	0.565	0.088	0.545	0.097
Tohana	0.598	0.071	0.950	0.096	0.403	0.075	0.459	0.080
Theoretical value of A^2 at 1% level is 1.057. Theoretical value of KS at 1% level is 0.210 for Fatehabad and Hansi, 0.243 for Hissar and 0.205 for Tohana.								

Analysis Based on Diagnostic Test

In addition to A^2 and KS tests, for identifying the best suitable distribution for estimation of rainfall, second line of action i.e., D-index was applied and these values were computed for EV1, EV2, LN2 and LP3 distributions using Eq. (3) and presented in Table 9.

From the diagnostic test results, as given in Table 9, it could be observed that the D-index value obtained from LN2 for Fatehabad, EV1 for Hansi and LP3 (MLM) for Hissar and Tohana were found as minimum.

Table9. *D-index values of EV1, EV2, LN2 and LP3 distributions*

Site	D-index values			
	EV1	EV2	LN2	LP3
Fatehabad	1.038	4.181	0.893	0.913
Hansi	3.395	8.596	4.549	3.915
Hissar	2.588	5.828	2.222	1.972
Tohana	1.762	12.882	1.762	1.604

Selection of Probability Distribution

Based on the findings obtained through GoF and diagnostic tests results, the study suggested LN2 is the most appropriate distribution for EVA of rainfall for Fatehabad whereas EV1 for Hansi, LP3 for Hissar and Tohana. The 100-year, 500-year and 1000-year return period estimated ER values with lower and upper confidence limits given by the selected distribution for the sites considered in the study are given in Table 10. The plots of estimated 1-day maximum rainfall by LN2 distribution for Fatehabad, EV1 for Hansi, LP3 for Hissar and Tohana together with the confidence limits and observed data are presented in Figures 1 to 4 respectively.

Table10. *Estimated ER values by selected probability distribution*

Site	Probability distribution	ER (mm) with 95% confidence limits		
		50-year	100-year	1000-year
Fatehabad	LN2	165.6 (124.6, 206.6)	192.0 (140.1, 243.9)	290.7 (192.5, 388.9)
Hansi	EV1	193.5 (149.6, 237.4)	221.1 (170.1, 272.1)	312.0 (236.7, 387.3)
Hissar	LP3	252.3 (149.2, 355.4)	295.0 (162.5, 427.5)	460.1 (197.5, 722.7)
Tohana	LP3	178.9 (124.4, 233.4)	201.3 (134.9, 267.7)	274.7 (163.4, 386.0)
Figures given in brackets indicate the lower and upper limits of the estimated ER.				

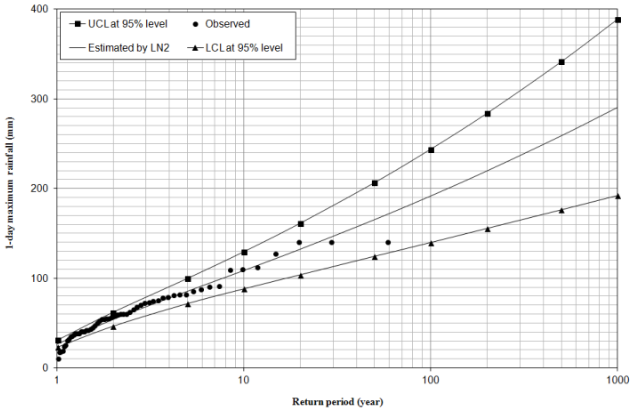

Figure1. *Estimated 1-day maximum rainfall by LN2 distribution with confidence limits and observed data for Fatehabad*

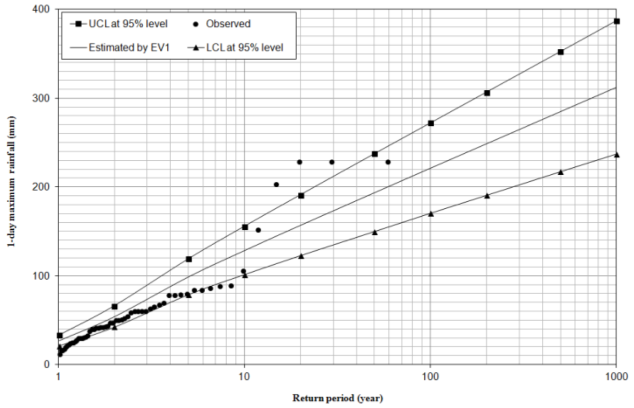

Figure2. *Estimated 1-day maximum rainfall by EV1 distribution with confidence limits and observed data for Hansi*

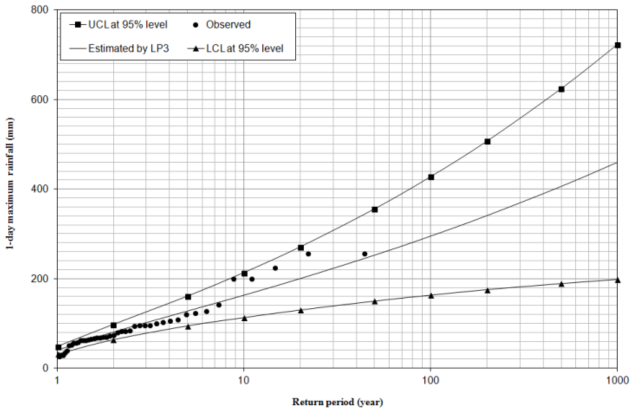

Figure3. *Estimated 1-day maximum rainfall by LP3 distribution with confidence limits and observed data for Hissar*

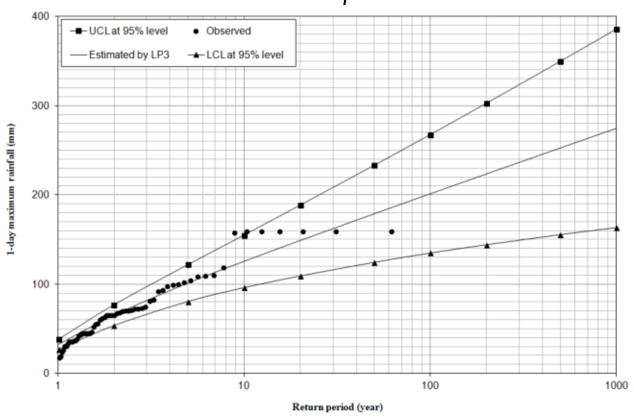

Figure4. *Estimated 1-day maximum rainfall by LP3 distribution with confidence limits and observed data for Tohana*

CONCLUSIONS

EVA of hydrometeorological parameters is essential in design consideration of establishment of hydraulic and civil structures. An effort is made to conduct study and evaluate the EV1, EV2, LN2 and LP3 distributions adopted in EVA of rainfall for Fatehabad, Hansi, Hissar and Tohana through GoF and diagnostic tests. The following conclusions were drawn from the study:

i) Analysis based on GoF tests results:

a) The A^2 test results suggested the applicability of LN2 and LP3 distributions for EVA of rainfall.

b) The A^2 test results ascertained the acceptability of EV1 distribution for Fatehabad and Tohana, and EV2 distribution for Hansi.

c) The KS test results suggested the applicability of EV1, EV2, LN2 and LP3 distributions for EVA of rainfall for all four sites.

ii) Qualitative assessment (plots of EVA results) of the outcomes was weighed with D-index values and accordingly LN2 was found to be acceptable for Fatehabad whereas EV1 for Hansi and LP3 for Hissar and Tohana.

iii) The study suggested that the upper limit of 1000-year return period estimated ER values of about 389 mm for Fatehabad, 387 mm for Hansi, 723 mm for Hissar and 386 mm for Tohana, could be considered for designing of hydraulic structures having design life of 1000-year.

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REFERENCES

[1] Naghavi, B., Yu, F.X. and Singh, V.P., 1993, "Comparative evaluation of frequency distributions for Louisiana extreme rainfall", *Water Resources Bulletin*, 29(2): 211-219.

[2] Di Balldassarre, G., Castellarin, A. and Brath, A., 2006, "Relationships between statistics of rainfall extremes and mean annual precipitation: an application for design-storm estimation in northern central Italy", *Hydrology and Earth System Sciences*, 10(2): 589–601.

[3] Mujere, N., 2011, "Flood frequency analysis using the Gumbel distribution", *Journal of Computer Science and Engineering*, 3(7): 2774-2778.

[4] Haberlandt, U., and Radtke, I., 2014., "Hydrological model calibration for derived flood frequency analysis using stochastic rainfall and probability distributions of peak flows", *Hydrology and Earth System Sciences*, 18(1): 353-365.

[5] Lee, C., 2005, "Application of rainfall frequency analysis on studying rainfall distribution characteristics of Chia-Nan plain area in Southern Taiwan", *Journal of Crop, Environment & Bioinformatics*. *2*(1): 31-38.

[6] Bhakar, S.R., Bansal, A.K., Chhajed, N., and Purohit, R.C., 2006, "Frequency analysis of consecutive days maximum rainfall at Banswara, Rajasthan, India", *ARPN Journal of Engineering and Applied Sciences*, 1(1): 64-67.

[7] Saf, B., Dikbas, F., and Yasar, M., 2007, "Determination of regional frequency distributions of floods in West Mediterranean River Basins in Turkey", *Fresenius Environment Bulletin*, 16(10): 1300–1308.

[8] Varathan, N., Perera, K., and Nalin., 2010, Statistical modelling of extreme daily rainfall in Colombo, Board of Study in Statistics and Computer Science of the postgraduate institute of science, *University of Peradeniya, Sri Lanka*.

[9] AlHassoun, S.A., 2011, "Developing empirical formulae to estimate rainfall intensity in Riyadh region", *Journal of King Saud University – Engineering Sciences*, 23(1): 81–88.

[10] Baratti, E., Montanari, A., Castellarin, A., Salinas, J.L., Viglione, A., and Bezzi, A., 2012, "Estimating the flood frequency distribution at seasonal and annual time scales", *Hydrological Earth System Science*, 16(12): 4651–4660.

[11] Esteves, L.S., 2013, "Consequences to flood management of using different probability distributions to estimate extreme rainfall", *Journal of Environ-mental Management*, 115(1): 98-105.

[12] Rasel, M., and Hossain, S.M., 2015, "Development of rainfall intensity duration frequency equations and curves for seven divisions in Bangladesh", *International Journal of Scientific and Engineering Research*, 6(5): 96-101.

[13] Zhang, J., 2002, "Powerful goodness-of-fit tests based on the likelihood ratio", *Journal of Royal Statistical Society*, 64(2): 281-294.

[14] Rao, A.R., and Hameed, K.H., 2000, Flood frequency analysis, CRC Press, Florida, USA.

[15] USWRC, 1981, Guidelines for deter-mining flood flow frequency, Bulletin No. 17B, United States Water Resources Council (USWRC).

[16] Weaver, B., 2002, Non-parametric tests, Probability and Hypothesis Testing.

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