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On the Vibration of Parallel Cylindrical Shells Under the Action of Harmonic Waves

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ABSTRACT

The paper deals with the stress-strain state of a parallel cylindrical tube with a liquid under the action of harmonic waves. The problem is solved in a bicylindrical coordinate system under the action of harmonic waves. An analytical solution is obtained in special Bessel and Hankel functions, as well as numerical results. Parametric analysis of the dynamic stress coefficient is carried out.

Keywords: *cylindrical tube, liquid, harmonic waves, bicylindrical coordinate system, special functions.*

INTRODUCTION

The problem of the action of elastic waves on single obstacles of simple form (cylindrical, spherical, and others) has been investigated [1-6], where the concentration of stress is carefully considered and the displacement field near the obstacle is studied closer. However, the effect of elastic (diffraction problems), the more viscoelastic waves on several obstacles, has not been fully investigated. In this paper, a methodology for investigating this problem has been developed. The need for the theory of diffraction and scattering by spatial and planar lattices has led to the appearance of a large number of papers on the diffraction of acoustic and electromagnetic waves on systems of scattering centers. A task of this type was considered by the authors of [7-9], but their approach cannot be directly extended to elastic waves. On the other hand, the method of multiple scattering, which is essentially an iterative process based on a systematic refinement of the results of the single-scattering approximation, was used in [10, 11]. This method can be used directly in problems on the propagation of elastic waves. In this paper we obtain a solution of the problem of the diffraction of a plane harmonic wave on parallel cylindrical layers with a liquid.

STATEMENT OF THE PROBLEM

Suppose that the viscoelastic medium is not constrained by the parallel arrangement of a cylindrical tube with a liquid (Fig. 1). The equation of viscoelasticity in vector form has the form[12,13]: $(\tilde{\lambda}_k+2\tilde{\mu}_k)grad\ div\ \bar{u}-\tilde{\mu}_k \mathrm{rotrot}\bar{u}=\rho_k\frac{\partial^2\bar{u}}{\partial t^2}$, к=1,2,3 (1)

where λ_k, μ_k - the Lame coefficients, defined by formulas

$$\tilde{\lambda}_k f(t)=\frac{\nu_k E_{0k}}{(1+\nu_k)(1-2\nu_k)}\left[f(t)-\int_{-\infty}^{t}R_{\lambda k}(t-\tau)f(t)d\tau\right],$$
$$\tilde{\mu}_k f(t)=\frac{\nu_k E_{0k}}{2(1+\nu_k)}\left[f(t)-\int_{-\infty}^{t}R_{\mu k}(t-\tau)f(t)d\tau\right], \qquad (2)$$

$\vec{u}$ - displacement vector, f(t)– derived time functions, $R_{\lambda k}(t-\tau)$, $R_{\mu k}(t-\tau)$ – relaxation core "to" body, $E_{0\kappa}$– instantaneous elastic modulus "to the" th body. The operators entering into equation [14] for the right system of curvilinear orthogonal coordinates are defined as follows

$$\mathrm{grad}\phi=\frac{1}{\sqrt{q_{11}}}\frac{\partial\phi}{\partial\alpha_1}\bar{i}_1+\frac{1}{\sqrt{q_{22}}}\frac{\partial\phi}{\partial\alpha_2}\bar{i}_2+\frac{1}{\sqrt{q_{33}}}\frac{\partial\phi}{\partial\alpha_3}\bar{i}_3,$$

$$\mathrm{rot}u=\frac{1}{\sqrt{q}}G$$

$$\mathrm{div}u=\frac{1}{\sqrt{q}}\left[\frac{\partial}{\partial\alpha_1}\left(u_1\sqrt{\frac{q}{q_{11}}}\right)+\frac{\partial}{\partial\alpha_2}\left(u_2\sqrt{\frac{q}{q_{22}}}\right)+\frac{\partial}{\partial\alpha_3}\left(u_3\sqrt{\frac{q}{q_{33}}}\right)\right],$$

$$G=\begin{vmatrix}\sqrt{q_{11}}\,\bar{i}_1 & \sqrt{q_{22}}\,\bar{i}_2 & \sqrt{q_{33}}\,\bar{i}_3\\ \frac{\partial}{\partial\alpha_1} & \frac{\partial}{\partial\alpha_2} & \frac{\partial}{\partial\alpha_3}\\ u_1\sqrt{q_{11}} & u_2\sqrt{q_{22}} & u_3\sqrt{q_{33}}\end{vmatrix},$$

where α_i- curvilinear coordinates (i = 1,3), q_{ij} - components of the metric tensor, determined by the formula: $q_{ij} = \sum_{k=1}^{3} \frac{\partial x_k}{\partial \alpha_i} \frac{\partial x_k}{\partial \alpha_j}$,x_k-Cartesian coordinates (k = 1,3), the q-square of the Jacobian of the transformation of the Cartesian coordinate system, and the curvilinear coordinate system. For orthogonal curvilinear coordinates, only the diagonal terms of the tensor matrix q_{ij} are not equal to zero. In this case $q = \sqrt{\prod_{i=1}^{3} q_{ii}}$, and the basic differential quadratic form is defined by the formula: $ds^2 = \sum_{i=1}^{3} q_{ii} d^2 \alpha_1$. To determine the stress state of the ground and the setting of mixed boundary conditions, it is necessary to have formulas expressing the stress through displacement. We use the geometric equations derived by Novitskii B.

$$\varepsilon_{ii} = \frac{\partial}{\partial \alpha_i}\left(\frac{u_i}{h_i}\right) + \frac{1}{2h_i^2} \sum_{j=1}^{3} \frac{\partial h_i^2}{\partial \alpha_j} \frac{u_j}{h_j} \qquad (3)$$

$$\varepsilon_{ij} = \frac{1}{2h_i h_j}\left[h_i^2 \frac{\partial}{\partial \alpha_i}\left(\frac{u_i}{h_i}\right) + h_j^2 \frac{\partial}{\partial \alpha_i}\left(\frac{u_j}{h_j}\right)\right]$$

$$i \neq j, \quad j = \overline{1,3}$$

In addition, we use the equation of state (Hooke's law) [15]

$$\sigma_{ij}^k = \tilde{\lambda}_k \varepsilon_{kk} \delta_{ij} + 2\tilde{\mu}_k \varepsilon_{ij} \quad (i,j,k = 1,2,3) \qquad (4)$$

As unknowns, we use the components of the displacement vector u_r, u_θ and u_z. The cylindrical coordinate system is related to the Cartesian coordinate system by the following relations:

x=rcosθ; y=rsinθ, z=z, $ds^2=dr^2+r^2d\theta^2+dz^2$. (5)

Using formula (5), we obtain

$$h_1^2 = h_3^2 = q_{11} = q_{33} = 1, \qquad h_{22}^2 = q_{22} = r^2$$

As coordinates α_i (i=1,3) applicable:

α_1=r, α_2=θ, α_3=z (6)

Substituting (5) and (6) in (1), and the resulting expression into formula (4) and taking into account, we obtain the following system of Lame equations in cylindrical coordinates:

$$\begin{aligned} &(\tilde{\lambda}+2\tilde{\mu})(u_r)_{rr} + \frac{\tilde{\mu}}{r}(u_r)_{\theta\theta} + \tilde{\mu}(u_2)_{zz} + \frac{\tilde{\lambda}+\tilde{\mu}}{r}(u_\theta)_{\theta r} + (\tilde{\lambda}+\tilde{\mu})(u_z)_{zz} + \\ &\quad + \frac{\tilde{\lambda}+2\tilde{\mu}}{r}(u_r)_r - \frac{\tilde{\lambda}+3\tilde{\mu}}{r^2}(u_\theta)_\theta - \frac{\tilde{\lambda}+2\tilde{\mu}}{r^r}u_r = \rho(u_r)_{tt}, \\ &\tilde{\mu}(u_\theta)_{22} + \frac{\tilde{\lambda}+2\tilde{\mu}}{r^2}(u_\theta)_\theta + \tilde{\mu}(u_\theta)_{zz} + \frac{\tilde{\lambda}+\tilde{\mu}}{r}(u_r)_{r\theta} + \frac{\tilde{\lambda}+\tilde{\mu}}{r}(u_z)_{z\theta} + \\ &\quad + \frac{\tilde{\mu}}{r^2}(u_r)_\theta - \frac{\tilde{\mu}}{r}u_\theta = \rho(u_\theta)_{tt}, \\ &\tilde{\mu}(u_z)_{rr} + \frac{\tilde{\mu}}{r^2}(u_\theta)_{\theta\theta} + (\tilde{\lambda}+2\tilde{\mu})(u_z)_{zz} + (\tilde{\lambda}+\tilde{\mu})(u_r)_{rr} + \\ &\quad + \frac{\tilde{\lambda}+\tilde{\mu}}{r}(u_\theta)_{\theta z} + \frac{\tilde{\lambda}+\tilde{\mu}}{r}(u_r)_z = \rho(u_z)_{tt} \end{aligned} \qquad (7)$$

Here the indices r, θ and z, behind the brackets denote the partial derivatives with respect to the corresponding coordinates. The boundary conditions along the outer surface of the pipe - the condition of ideal contact with the ground, the inner surface is free:

$$\begin{aligned} &r = R: u_{r1} = u_{r2}, u_{\theta 1} = u_{\theta 2}, u_{z1} = u_{z2}, \\ &\sigma_{rr1} = \sigma_{rr2}, \sigma_{r\theta 1} = \sigma_{r\theta 2}, \sigma_{rz1} = \sigma_{rz2}, \\ &r = R_0: \sigma_{rr2} = 0, \sigma_{r\theta 1} = 0, \sigma_{rz1} = 0, \end{aligned} \qquad (8)$$

where the subscripts "1" and "2" denote respectively the materials of the environment and the pipe. If in a cylindrical tube with a liquid, then the boundary conditions ensuring the equality of the normal components of the fluid and shell velocities are

$$(\vec{v}\,\vec{n})\Big|_{r=R_0} = +\frac{\partial u_{r2}}{\partial t}, \qquad (9)$$

where $\vec{v}$ - fluid particle velocity; $\vec{n}$ - normal surface at $r=a$, w- radial displacement of the shell, $D = 2R_0$. In order to completely complete the formulation of the problem, it is necessary to add conditions at infinity to conditions (8) and (9) $u \to 0$

at $R = \sqrt{x^2+y^2+z^2} \to \infty$ (10)

For non-stationary problems, the causality principle is required as the radiation conditions, and in the medium there should be no displacements outside the region bounded by the leading edge of the waves from the oscillation sources.

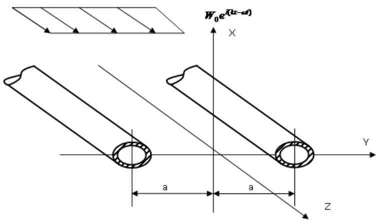


Fig1. *The calculation scheme*

METHODS OF SOLUTION

We consider the problem of the dynamic theory of linear elasticity, the effect of seismic waves on pipes stacked in a high embankment in two strands and filled with an ideal compressible fluid. In this case, let us consider the case when the wave falls perpendicular to the axis connecting the tube centers, and to the longitudinal axis of these tubes. The calculation scheme is shown in Fig. The bicylindrical coordinate system is related to the Cartesian coordinate system by the following relations:

x=(a shξ)/(chξ- cosη), y=(a sinη)/(chξ - cosη), z=z (11)

where: a - half the distance between points η=-∞ и η=∞.

Then, representing (11) in (5), and the resulting expressions in (6) take the following form:

$$ds^2 = a^2(\mathrm{ch}\eta - \cos\xi)^{-2}\, d\xi^2 + a^2(\mathrm{ch}\eta - \cos\xi)^{-2} d\eta^2 + dz^2 \quad (12)$$

Using formula (11), we obtain

$$h_1^2 = h_2^2 = q_{11} = q_{22} = a^2(\mathrm{ch}\eta - \cos\xi)^{-2},$$
$$h_3^2 = q_{33} = 1. \qquad (13)$$

Assuming that: $\alpha_1=\xi$, $\alpha_2=\eta$, $\alpha_3=z$ and substituting (12) and (13) in (1) - (11), and, taking into account that the problem is flat, we obtain the following Helmholtz equation in bipolar coordinates:

$$\left[a^{-2}(\mathrm{ch}\eta - \cos\xi)^2\right]\left[(v)_{\xi\xi} + (v)_{\eta\eta}\right] + k^2 v = 0 \quad (14)$$

where

$$\frac{\sin\xi}{\mathrm{ch}\eta - \cos\xi} = \begin{cases} 2\sum_{n=1}^{\infty} e^{-n\eta}\sin n\xi & \eta > 0 \\ 2\sum_{n=1}^{\infty} e^{n\eta}\sin n\xi & \eta < 0 \end{cases} \quad (15)$$

Equation (14), after certain transformations, reduces to the form in

$$(v)_{\xi\xi} + (v)_{\eta\eta} + (2kae^{\pm\eta})^2 v = 0 \qquad (16)$$

We seek the solution of equation (14) in the form of a series:

$$v = \sum_{n=0}^{\infty}\left[v_n^a(\eta)\cos n\xi + v_n^b(\eta)\sin n\xi\right]\varepsilon^{-iwt} \qquad (17)$$

Substituting (17) into (16) and equating the coefficients for the corresponding harmonics, we obtain the following ordinary differential equation:

$$v_n'' + \left[(2kae^{\pm\eta})^2 - n^2\right]v_n = 0 \qquad (18)$$

Standard replacement

$$v_n(\eta) = z(t) \quad , t=\exp(\pm\eta)$$

we reduce (18) to the Bessel equation of the form

$$t^2 z'' + tz' + (4k^2a^2 - n^2)z = 0 \qquad (19)$$

which has a particular solution in the form of a cylindrical function z(2ake$^{-\eta}$), and the solution of the Helmholtz equation takes the following form:

$$\phi = \sum_{n=0}^{\infty} A_n Z_n(2ake^{\mp\eta})\cos n\xi e^{-i\omega t},$$

$$\psi = \sum_{n=0}^{\infty} B_n Z_n(2ake^{\mp\eta})\sin n\xi e^{-i\omega t}. \qquad (20)$$

Now we put the boundary conditions. To this end, we use condition (20), the substitution r=η и θ=ξ. Taking into account the obtained relations, we will seek the solution of the boundary value problem for the case of a fall in two underground pipes of the P-wave of compression and the SV-wave of the shift perpendicular to the y-axis. The wave potential wave has the form

$$\phi^{(p)} = Ae^{i\alpha x - i\omega t}. \qquad (21).$$

To represent (21) in the form (20), we write (21) with the aid of (12) in bipolar cylindrical coordinates.

$$\phi_1^{(p)} = Ae^{ik2a\exp(\mp)^{\eta}\sin\xi e^{-i\omega t}}. \qquad (22)$$

Expanding the second factor of expression (22) into a Fourier series (complex form) and after small transformations we obtain the final expression for the potential of the incident P wave:

$$\phi_1^{(p)} = A\sum_{n=0}^{\infty}\varepsilon_n J_n(\alpha_1\tau)\cos n\xi e^{-i\omega t} \qquad (23)$$

where τ=2aexp(∓η) and for the potential of the incident SV-wave:

The remaining potentials (20), in analogy with (23), have the form:

$$\phi_2^{(r)} = \sum_{n=0}^{\infty}\left[C_n H_n^{(1)}(\alpha_2\tau) + D_n H_n^{(2)}(\alpha_2\tau)\cos n\xi e^{-i\omega t}\right],$$

$$\psi_2^{(r)} = \sum_{n=0}^{\infty}\left[E_n H_n^{(1)}(\beta_2\tau) + F_n H_n^{(2)}(\beta_2\tau)\sin n\xi e^{-i\omega t}\right], \qquad (24)$$

$$\phi_3^{(r)} = \sum_{n=0}^{\infty} G_n J_n^{(1)}(\alpha_3 \tau)\cos n\xi e^{-i\omega t}.$$

Dynamic VAT is expressed in terms of potentials φ_1 and ψ_2:

$$u_{\eta k} = \delta\left[(\phi_k)_\eta - (\psi_k)_\xi\right],$$

$$u_{\eta 3} = -\delta(iw)^{-1}(\phi_3),$$

$$u_{\xi n} = \delta\left[(\phi_n)_\xi - (\psi_n)_\eta\right], \qquad (25)$$

$$\sigma_{\eta\eta n} = -\sigma_{\xi\xi n} = 2\delta^2\left\{d_n\left[0{,}5\phi_{\eta\eta} - (\phi_\xi + \phi_\eta)\sin\xi\right] + 0{,}5\lambda_n\phi_{\xi\xi} - \mu_n(\psi_{\xi\xi} - \phi_\eta + \psi_\xi)\right\},$$

$$\tau_{\eta\eta 3} = \sigma_{\xi\xi 3} = -iw_3\rho_3\phi_3,$$

$$\tau_{\eta\xi n} = 2\mu_n\delta^2\left[\phi_{\xi\eta} + 0{,}5\psi_{\eta\eta} - 0{,}5\psi_{\xi\xi} + \phi_\xi + \psi_\eta + (\phi_\xi - \psi_\xi)\sin\xi\right],$$

$$n = 1,2; \delta = e^{\mp\eta}/2a.$$

Substituting (24) and (25) in (8) we obtain the final solutions to the problems of the fall of the P and SV waves respectively on two underground pipes. Arbitrary fixed (A_n, B_n, C_n and others) are determined from a system of algebraic equations with complex coefficients

$$[C]\{q\}=\{\rho\}. \qquad (26)$$

where C is the determinant of (12x12) -order, the elements of which are the Bessel and Hankel functions of the 1 st second kind of the n-th order, q is the column vector of unknown quantities, and ρ is the vector of the right-hand side. A system of algebraic equations with complex coefficients is solved by the Gauss method with the separation of the principal element. Dynamic VAT in case of a fall-wave shift to two underground pipes is also recorded in bipolar coordinates in an asymptotic form:

$$u_z = w, \sigma_{\eta z} = \mu_i\delta(u_z)_\eta, \sigma_{\xi z} = \mu_i\delta(u_z)_\xi$$

As boundary conditions, we use condition (23) and the substitution r = n. The final solution of the problem for the cases of the incidence of an SH-wave on two tubes is:

$$u_{z2} = -w_0\sum_{n=0}^{\infty}\left[B_n H_n^{(1)}(k_2\tau) + C_n H_n^{(2)}(k_2\tau)\right]\cos n\xi e^{-i\omega t};$$

$$\sigma_{rz2} = -\mu_2 w_0 k_2\sum_{n=0}^{\infty}\left[B_n H_n^{(1)}(k_2\tau) + C_n H_n^{(2)}(k_2\tau)\right]\cos n\xi e^{-i\omega t};$$

$$\sigma_{\theta z2} = \mu_2 w_0 n\sum_{n=0}^{\infty}\left[B_n H_n^{(1)}(k_2\tau) + C_n H_n^{(2)}(k_2\tau)\right]\sin n\xi e^{-iwt}; \qquad (27)$$

$$u_{z1} = w_0\sum_{n=0}^{\infty}\left[\varepsilon_n J_n(k_1\tau) + A_n H_n^{(1)}(k_1\tau)\right]\cos n\xi e^{-i\omega t};$$

$$\sigma_{rz1} = \mu_1 w_0 k_1\sum_{n=0}^{\infty}\left[\varepsilon_n J_n(k_1\tau) + A_n H_n^{(1)}(k_1\tau)\right]\cos n\xi e^{-i\omega t};$$

$$\sigma_{\theta z1} = -\mu_1 w_0 n\sum_{n=0}^{\infty}\left[\varepsilon_n(k_1\tau) + A_n H_n^{(1)}(k_1\tau)\right]\sin n\xi e^{-i\omega t};$$

Uncertain coefficients A_n,B_n,C_n is determined from the boundary conditions. Consider the definition of the dynamic stress-strain state of a cylindrical tube under the action of harmonic waves. To solve the problem, an addition theorem is applied. The addition theorems for cylindrical wave functions are derived in [4,5,6]. Let there be two different polar coordinate systems (r_g,θ_g) and (r_k,θ_k) (Fig. 3), in which the polar axes are equally directed. Pole coordinate θ_k in the q system will be R_{kq}, θ_{kq}, so that equality

$$Z_g = R_{kg}e^{i\theta_{kg}} + Z_k \qquad (28)$$

Then the addition theorem has the form:

$$b_n(\alpha r_q)e^{in\theta_q} = \sum_{p=-\infty}^{\infty} b_{n-p}(\alpha R_{kq})e^{i(n-p)\theta_{kq}}Tp(\alpha r_k)\exp(ip\theta_k),\ r_k < R_{kq},$$

$$b_n(\alpha r_q)e^{in\theta_q} = \sum_{p=-\infty}^{\infty} J_{n-p}(\alpha R_{kq})e^{i(n-p)\theta_{kq}}b_p(\alpha r_k)\exp(ip\theta_k),\ r_k < R_{kq} \qquad (29)$$

Formula (28) makes it possible to transform the solution of the wave equation (1) from one coordinate system to another. Consider the calculation of an extended underground multi-thread pipeline for seismic action within the framework of the plane problem of the dynamic theory of elasticity. In this case, we investigate the case of stationary diffraction of plane waves on a series of periodically located cavities, supported by rings with an ideal compressible fluid inside. The solution of the problem is realized by the method of potentials. The boundary conditions have the form (8). The form of the incident potential will not change either. The potentials of the waves reflected from the tubes after the application of the addition theorem, and taking into account the periodicity of the problem, will have the form:

$$\phi_1^{(r)} = e^{-i\omega t}\sum_{n=0}^{\infty}\left[A_n H_n^{(1)}(\alpha_1 r) + S_n J_n(\alpha_1 r)\right]e^{in(\theta-\gamma)},$$

$$\psi_1^{(r)} = e^{-i\omega t}\sum_{n=0}^{\infty}\left[B_n H_n^{(1)}(\beta_1 r) + \sigma_n J_n(\beta_1 r)\right]e^{in(\theta-\gamma)},$$

$$S_n = \sum_{p=0}^{\infty}\sum_{m=1}^{\infty} A_p E_p \left[e^{im\xi} H_{n-p}^{(1)}(\alpha_1 m\delta) + e^{-im\xi} H_{n-p}^{(1)}(\alpha_1 m\delta) \right], \quad (30)$$

$$Q_n = \sum_{p=0}^{\infty}\sum_{m=1}^{\infty} B_p E_p \left[e^{im\xi} H_{n-p}^{(1)}(\beta_1 m\delta) + e^{-im\xi} H_{n-p}^{(1)}(\beta_1 m\delta) \right],$$

where: $\xi=k\delta\cos\gamma, \delta$ - distance between pipe centers.

The refracted wave potentials in the tubes are written in the form

$$\varphi_2 = e^{i(m\xi-\omega\xi)} \sum_{n=0}^{\infty} E_n \left[C_n H_n^{(1)}(\alpha_1 r) + D_n H_n^{(2)}(\alpha_2 r) \right] e^{in(\theta-\gamma)},$$

$$\psi_2 = e^{i(m\xi-\omega\xi)} \sum_{n=0}^{\infty} E_n \left[E_n H_n^{(1)}(\beta_1 r) + F_n H_n^{(2)}(\beta_2 r) \right] e^{in(\theta-\gamma)}, \quad (31)$$

and the velocity potential in the ideal form of a compressible fluid

$$\varphi_3 = e^{i(m\xi-\omega\xi)} \sum_{n=0}^{\infty} E_n G_n J_n(\alpha_3 r) e^{in(\theta-\gamma)}, \quad (32)$$

Unknown coefficients A_n-G_n are determined by setting (29) - (32) in (8). As a result, an infinite system of linear equations is obtained, which is solved by an approximate reduction method, provided that relation

$$k\delta(1 \mp \cos\gamma) = 2\pi n$$

The general characteristic of the program is designed for multi-threaded pipes in the embankment for the case of a drop in seismic waves perpendicular to the axis passing through the pipe centers. The information entered contains the minimum necessary data: the elastic characteristics (E and ν) of the soil of the embankment and the pipes; density of soil, pipes and liquids filling it; internal and external pipe radii; the predominant period of oscillation of soil particles; coordinates of the point where the VAT is located; seismic coefficient.

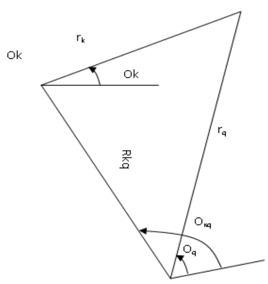


Fig2. *Scheme to the addition theorem.*

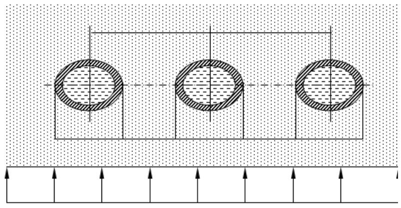
With the help of a special label, it is possible to calculate pipes filled with an ideal compressible fluid, or empty ones. The calculation of the cylindrical Bessel and Hankel functions is carried out according to known formulas. The solution of the system of linear equations is carried out by the Gauss method with the separation of the principal term.

NUMERICAL RESULTS AND DISCUSSIONS

Effect of distance between pipes

Table 1 shows the values of the coefficient $\eta_{max} (\eta_{max} = |\sigma_{rr}| / (\lambda + 2\mu)\alpha^2 A$

the maximum radial pressure of the soil on the pipes at different distances d between them in the event of a P-wave fall. In this case, the wave number of the P wave α_r=1,0: inner and outer radius of pipes R_0=0,8 m and R=1,0 m: the predominant period of oscillation of soil particles is T = 0.2 sec. Soil Characteristics: Permanent Lamé λ_1=8,9-MPa; μ_1=4,34MPa; density ρ_1=1,74Kn sek^2/m^4.



$$\varphi = Ae^{i(kx-\omega t)}$$

Fig3. *The calculation scheme*

Pipe material characteristics λ_2=8690MPa; μ_2=12930MPa; ρ_2=2,55Kn sek^2/m^4. We take the following parameters:

$A = 0{,}048$; $\beta = 0{,}05$; $\alpha = 0{,}1$. From tables 1 it follows that as the distance between the pipes 0,5≤d/D≤1,0 coefficient η_{max} increases by 5%. And with a further increase, d / D> 1.0 decreases more sharply by 10%. For d / D> 2.0, the value η_{max} stabilizes, i.e. practically does not change, with l≤4,0 close to the value η_{max} for a single pipe according to calculations. Consequently, the mutual influence of reinforced concrete pipes of multiline stacking takes place with the distance between them d≤4,0D and leads to an increase in the maximum dynamic pressure of the soil on them compared to a single pipe. This effect of increasing the coefficient η_{max} is associated with the imposition of waves reflected by several surfaces of multicell pipes. In this case, the nonmonotonic increase in the coefficient η_{max} with a decrease in the distance between the tubes d / D is connected in our opinion with the phenomenon of interference superimposed after reflection of the waves.

Table1. *The value of the coefficient of dynamic concentration at different distances between the tubes for the case of P-wave incidence*

D/d		0,5		1,0		2,0		4,0
η_{max}		1,68		1,76		1,61		1,60

This phenomenon is extremely important for the practice of designing seismic underground multiline pipelines, since allows you to choose the optimal distance between the pipes, in which the dynamic pressure during seismic action is minimal. For example, in tables 1, such a distance is d = 0.5D. It is well known to note, for comparison, that in the case of static action, the reverse picture is observed: the ground pressure on multicell pipes is less than that for a single one. In addition to the above, when analyzing the influence of the distance between pipes on their VAT, it is necessary to take into account the relation (28), (the so-called "slip points"), at which a significant increase in the dynamic stresses in the vicinity of the tube-resonance is observed. This phenomenon is known from optics called Wood's anomaly is a feature of the multi-threaded pipeline and cannot arise in a pipeline laid in one thread. From the point of view of design practice, it is necessary to know at what distance it is possible to lay pipes so that a dangerous phenomenon of resonance does not arise. The answer to this question is given by the relation (27). Let us analyze this relation for the case of the action of P and SV seismic waves on a subterranean pipeline. Tables 2 show the dependence of the maximum distance in the light between the centers of the pipes d_{max}, at which there is no resonance, from the angle of incidence of seismic waves γ.

Table2. *Dependence of distance D_{max} from the angle of incidence γ.*

γ. рад		0	30	45	60	70	80		90
D_{max}, м		5, 0	5, 36	5, 86	6, 66	7, 45	8, 52		10 ,0

From tables 2 it follows that the smaller the angle of incidence of the seismic wave on the pipeline, the closer it is to each other to stack pipes. Thus, the appearance of resonance in multi-threaded pipes can be avoided by choosing the appropriate distance between them and, thereby, ensuring the seismic stability of the pipeline. Influence of the type of seismic action (P-, SV- or SH-wave). Tables 3 give the values η_{max} the maximum radial pressure of the soil on the pipes in the event of a fall in the P- and SV-seismic waves at different distances d between the pipes.

At the same time, β_r=2. Analysis of the data in Tables 3 shows that for d / D <4.0, the coefficient values $\eta_{max,}$ for P-wave and SV-wave are as if in antiphase. That eats, at l / D = 1.0, the maximum seismic effect of the P-wave is 27% higher than that of the SV wave. At d / D = 2.0, the maximum seismic effect of the P-wave is 7% lower than that of the SV wave, and at d / D = 4.0 again higher, but only 1%. At the same time, as the distance between the pipes increases, the difference in these effects decreases and at d / D = 4.0 it practically disappears altogether. In addition, we note that when an SV-wave is applied, the values η_{max} at different distances between the pipes has a 2.5 times greater spread (up to 25%) than when the P wave is applied (up to 10%). Thus, the phenomenon of "local resonance" manifests itself more strongly for seismic action in the form of an SV wave.

Table3. *Coefficient value η_{max} with seismic actions in the form of P and SV waves at different distances d between the pipes*

d/D	η_{max}		
	P – wave		**SV - wave**
1,0	1,76		1,29
2,0	1,61		1,72
4,0	1,60		1,51

Influence of fluid filling pipes

Tables 4 show the values of the coefficient η_{max} in the case of a fall of P-wave on empty and water-filled pipes at different distances d between the pipes. The density of the liquid was assumed equal to ρ_3=0,102Kn sek^2/m^4. From Table 4 it follows that the presence of water in the pipes increases seismic effects on them compared to empty pipes. Obviously, this is due to the increase in the weight of the pipeline. The maximum dynamic pressure of the soil on the pipes is enhanced.

For example: for d / D = 1.0, the difference in the values of the coefficient d / D = 2.0-10%, with d / D = 4.0-19%. In addition, we note that the spread of the coefficient values η_{max} at different distances d pipes filled with water are less (7%) than for empty pipes (10%).

Table4. *Coefficient value η_{max} for the case of the fall of P-wave on empty and water-filled pipes*

d/D	η_{max}	
	P - wave	**SV - wave**
1,0	1,76	1,89
2,0	1,61	1,78
4,0	1,60	1,90

Influence of the length of the incident seismic wave

Tables 4 show the values of the coefficient η_{max} different lengths l_0/l_0-$2\pi/\alpha$, p - wave incident on empty pipes, located at a distance l = 1,0D from each other.

Table5. *Values of the coefficient η_{max} for different lengths l_0 P - waves.*

l_0/D	3,0		5,0		10,0
η_{max}	1,76		1,52		1,20

From tables 5 it follows that the greater the length of the incident seismic wave, i.e. The denser the soil of the embankment, the lower the coefficient η_{max}. For reference, we note that relation l_0/D=5,0 – not in bulk sand, sandy loam and clayey soils; l_0/D=10,0 - clay soils. Thus, the type of soil, and especially its density, has a significant effect on its dynamic pressure on the pipes under seismic action. Hence it follows that when erecting a mound over pipes, it is necessary to carefully compact the bulk ground. It is interesting to note that a good compaction of the soil can also reduce its static pressure on the pipes. In addition, the calculations show that when l_0>10,0D The dynamic problem reduces to a quasistatic problem, which essentially simplifies its solution. From this follows the important conclusion that the quasistatic approach is not applicable to the calculation of the seismic effect of pipes under embankments.

Effect of wall thickness of pipe

Tables 6 give the coefficients η_{max} for different thickness t of the reinforced concrete pipe wall in the case of a P-wave fall onto empty multi-threaded pipes, stacked multi-threaded pipes laid at a distance d=0,5.

Table6. *Coefficient value η_{max} for different pipe wall thicknesses t*

d/D		0,08	0,1		0,15		0,2
η_{max}		1,60	1,66		1,66		1,68

From Tables 6 it follows that the range of wall thickness, practically does not affect the dynamic pressure of the soil, not these pipes. This is most likely due to the fact that the harmonic wave does not penetrate the reinforced concrete pipe due to the sufficient rigidity of the pipe.

Conclusions

- Under the harmonic (seismic) effect, the mutual influence of reinforced concrete pipes of multiline stacking takes place at a distance d> 4,0D
- $(d = 2a)$ between them and leads to an increase in the maximum dynamic pressure of the ground on them as compared to a single pipe (local resonance phenomenon) by 5-10% .
- The appearance of resonance in multicell pipes can be avoided if the distance between them is chosen not to the length of the incident harmonic wave. This phenomenon of resonance is a feature of the multi-threaded pipeline and cannot arise in a pipeline laid in one thread. The phenomenon of local resonance manifests itself more strongly for seismic action in the form of SV-wave than P-waves.
- The denser the soil of the embankment, the less seismic impact on underground pipes. When *l*>10D (wavelength) the dynamic problem is reduced to quasistatic. Also, the change in wall thickness and concrete class practically does not affect the dynamic pressure of the soil on reinforced concrete pipes under seismic action.
- Maximum dynamic ground pressure σ_{max} on pipes laid in two strings at a distance d <3.0D from each other, more than a single pipe. This excess reaches 15%.
- The presence of liquid in the pipes, as a rule, increases the pressure σ_{max} for a single pipe by 20% and for two thread pipes by 5-10%. The exception is densely packed pipes d = 0, for which the pressure σ_{max} decreases by 4%.

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