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Cross Diffusion Effects on Thermal Instability in a Maxwell Fluid Saturated Rotating Anisotropic Porous Medium

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ABSTRACT

Linear and nonlinear stability analyses are performed to study the double diffusive convection in a Maxwell fluid saturated rotating anisotropic porous layer in the presence of the Soret and Dufour effects. Linear stability analysis has been performed by using normal mode technique, while nonlinear analysis is done using truncated Fourier series. The flow is also affected by temperature and concentration gradients in their medium. The modified Darcy model has been employed in the momentum equation. Effects of mechanical anisotropy parameter, relaxation parameter, retardation parameter, Darcy-Prandtl number, Dufour parameter, Soret parameter, solute Rayleigh number and Lewis number on the stationary and oscillatory modes of convection have been obtained and are shown graphically. Further, heat and mass transports across the porous medium are also presented in the figures.

Keywords: *Maxwell Fluid; Double Diffusive Convection; Dufour Parameter; Soret Parameter; Porous Medium.*

INTRODUCTION

Anisotropy is generally a consequence of preferential orientation of asymmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature. Anisotropy is particularly important in a geological context, since sedimentary rocks generally have a layered structure; the permeability in the vertical direction is often much less than in the horizontal direction. Anisotropy can also be a characteristic of artificial porous materials like pelleting used in chemical engineering process and fiber material used in insulating purpose. The review of research on convective flow through anisotropic porous medium has been well documented by McKibbin [1] and Storesletten [2, 3]. Bhadauria et al. [4] have studied natural convection in a rotating anisotropic porous layer with internal heat source. Gaikwad et al. [5] performed linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in the presence of Soret effect.

Malashetty and Kollur [6] investigated the onset of double diffusive convection in a couple stress fluids saturated anisotropic porous layer. Gaikwad et al. [7] have carried out cross-diffusion effects on the onset of double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer.

The studies of double diffusive convection in porous media play very significant roles in many areas such as in petroleum industry, solidification of binary mixture, migration of solutes in water saturated soils. Other examples include geophysics system, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, Earth's oceans, magma chambers etc. The problem of double diffusive convection in a porous media has been presented by Ingham and Pop [8], Nield and Bejan [9], Vadasz [10] and Vafai [11, 12]. Very first study on double diffusive convection in porous media mainly concerns with linear stability analysis, was performed by Nield [13]. In the presence of cross diffusion one comes across two effects; Soret effect and Dufour effect. Soret effect, which describes the tendency of a salute to diffuse under the influence of a temperature gradient.

Similarly, a flux of heat caused by a spatial gradient of concentration is called the Dufour effect. There are many studies available on the effect of cross-diffusion on the onset of double-diffusive convection in a porous medium. Thermal convection in a binary fluid driven by the Soret and Dufour effects has been investigated by Knobloch [14] (1980).

He has shown that equations are identical to the thermosolutal problem except regarding a relation between the thermal and solute Rayleigh numbers. Hurle and jakeman [15] and Straughan and Hutter [16] argued that the Dufour coefficient is of order of magnitude smaller than the Soret coefficient in liquids, and the corresponding contribution to the heat flux may be neglected. The double-diffusive convection in a porous medium in the presence of Soret and Dufour coefficients has been studied by Rudraiah and Malashetty (1986) [17] for a Darcy porous medium using linear analysis, which was extended to include weak nonlinear analysis by Rudraiah and Siddheshwar [18]. Linear and nonlinear study of double diffusive convection in a fluid saturated porous layer with cross-diffusion effects has been carried out by Malashetty and Biradar [19]. The onset of double diffusive convection in a binary Maxwell fluid saturated porous layer with cross-diffusion effects has been carried out by Malashetty and Biradar [20]. Gaikwad and Dhanraj [21] analyses the onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer with cross-diffusion effects.

Recently, as some new technologically significant materials are discovered acting like non-Newtonian fluids, therefore, mathematicians, physicists and engineers are actively conducting research in rheology. Maxwell fluids can be considered as a special case of a Jeffreys-Oldroyd B fluid, which contain relaxation and retardation time coefficients.

Maxwells constitutive relation can be recovered from that corresponding to Jeffreys-Oldroyd B fluids by setting the retardation time to be zero. Several fluids such as glycerin, crude oils or some polymeric solutions, behave as Maxwell fluids. Viscoelastic fluid flow in porous media has attracted considerable attention due to the large demands of such fluids in diverse fields such as biorheology, geophysics, chemical industries, and petroleum industries.

Wang et al. [22] have made the stability analysis of double diffusive convection of Maxwell fluid in a porous medium. It is worthwhile to point out that the first viscoelastic rate type model, which is still used widely, is due to Maxwell. The onset of double diffusive convection in a viscoelastic saturated porous layer has been considered by many researchers [23]-[27]. Bhadauria et al. [28] have studied nonlinear two dimensional double diffusive convection in a rotating porous layer saturated by a visco elastic fluid. Narayana et al. [29] performed linear and nonlinear stability analysis of binary Maxwell fluid convection in a porous medium.

Recently, Zhao at al. [30] have done linear and nonlinear stability analysis of double diffusive convection in a Maxwell fluid saturated porous layer with internal heat source. Gaikwad et al. [31] have studied onset of Darcy-Brinkman convection in a Maxwell fluid saturated anisotropic porous layer. Ram et al. [32] have studied onset instability of thermosolutal convective flow of visco elastic Maxwell fluid thorough porous medium with linear heat source effect. More recently, Awad et al. [33] used the Darcy-Brinkman-Maxwell model to study linear stability of a Maxwell fluid with cross-diffusion and double-diffusive convection. They found that the effect of relaxation time is to decrease the critical Darcy-Rayleigh number. Altawallbeh et al. [34] have done linear stability analysis of double diffusive convection in a viscoelastic fluid saturated porous layer with cross diffusion effects and internal heat source.

However, to the best of author's knowledge no literature is available in which linear and nonlinear stability analysis for cross diffusive convection has been done in an anisotropic porous media saturated with viscoelastic fluid of type Oldroyd-B. Therefore, in the present study stability analysis of Maxwell fluid saturated rotating anisotropic porous layer has been done on double diffusive convection in the presence of cross diffusion. Our objective is to study how the onset criterion for oscillatory convection is affected by Maxwell fluid and the other parameters, and also to find heat and mass transports. In the limiting cases, some previously published results can be recovered as the particular cases of our results.

MATHEMATICAL FORMULATION

Consider an infinitely extended horizontal anisotropic porous layer saturated with Maxwell fluid mixture, confined between two planes at $z=0$ and $z=d$ heated from below and cooled from above. Darcy model that includes the

Coriolis term is used for the momentum equation. A Cartesian frame of reference is chosen with x -and y -axes at the lower boundary plane and z -axis directed vertically upwards in the gravity field.

The axis of rotation is assumed to coincide with the z -axis, rotating with a constant angular velocity Ω.

An adverse temperature gradient is applied across the porous layer. The lower planes is kept at temperature $T_0 + \Delta T$, while upper planes is kept Temperature T_0, with concentration $S_0 + \Delta S$ and S_0 respectively. The governing equations are as given below

$$\nabla.\vec{q} = 0 \tag{1a}$$

$$\left(1+\overline{\lambda}_1 \frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q}{\partial t} + 2\frac{\rho_0}{\varepsilon}\ \Omega\times\vec{q}\right) = \left(1+\overline{\lambda}_1 \frac{\partial}{\partial t}\right)\left(-\nabla p + \rho g\right) + \mu\left(K.\,\vec{q}\right) \tag{1b}$$

$$\gamma\frac{\partial T}{\partial t} + \left(\vec{q}.\nabla\right)T = \kappa_T \nabla^2 T + K_{TS}\nabla^2 S, \tag{1c}$$

$$\varepsilon\frac{\partial S}{\partial t} + \left(\vec{q}.\nabla\right)S = \kappa_S \nabla^2 S + K_{ST}\nabla^2 T, \tag{1d}$$

$$\rho = \rho_0\left[1-\beta_T\left(T-T_0\right) + \beta_S\left(S-S_0\right)\right] \tag{1e}$$

Where the physical variables have their usual meanings as given in the nomenclature. The externally imposed the thermal and solutal boundary conditions are as given by

$$\begin{aligned} &T = T_0 + \Delta T, \quad at \quad z=0 \ \ and \ T = T_0, \quad at \ \ z = d, \\ &S = S_0 + \Delta S, \quad at \quad z=0 \ \ and \ \ S = S_0, \quad at \ \ z = d, \end{aligned} \tag{2}$$

Basic Solution

At this state the velocity, pressure, temperature, concentration and density profiles are given by

$$q_b = 0, \quad p = p_b(z), \quad T = T_b(z), \quad S = S_b(z), \quad \rho = \rho_b(z). \tag{3}$$

Substituting Eq. (3) in Eq. (1a-1e), we get the following equations are obtained

$$\frac{dp_b}{dz} = -\rho_b g, \tag{4}$$

$$\frac{d^2 T_b}{dz^2} = 0, \tag{5}$$

$$\frac{d^2 S_b}{dz^2} = 0, \tag{6}$$

$$\rho_b = \rho_0\left[1-\beta_T(T_b - T_0) + \beta_S(S_b - T_0)\right]. \tag{7}$$

Now, we superimpose infinite amplitude perturbations on the basic state in the form:

$$q = q_b + q', \quad T = T_b + T', \quad p = p_b + p', \quad S = S_b + S', \quad \rho = \rho_b + \rho', \tag{8}$$

Substituting Eqs.(8) into Eqs. (1a) -(1e), using Eqs. (4) - (7), to get

$$\nabla.q' = 0 \tag{9}$$

$$\left(1+\overline{\lambda}_1 \frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q'}{\partial t} + 2\frac{\rho_0}{\varepsilon}\ \Omega\times\vec{q}'\right) + \mu\left(K.\vec{q}'\right) = \left(1+\overline{\lambda}_1 \frac{\partial}{\partial t}\right)\left(-\nabla p' - \rho(\beta_T T' - \beta_S S')g\right) \tag{10}$$

$$\gamma\frac{\partial T'}{\partial t} + \left(q'.\nabla\right)T' + w'\frac{\partial T_b{}'}{\partial z} = \kappa_T \nabla^2 T' + K_{TS}\nabla^2 S' \tag{11}$$

$$\varepsilon\frac{\partial S'}{\partial t}+\left(q'.\nabla\right)S'+w'\frac{\Delta S}{d}=\kappa_S\nabla^2 S'+K_{ST}\nabla^2 T' \tag{12}$$

$$\rho'=-\rho_0\left[\beta_T T'+\beta_S S'\right] \tag{13}$$

The pressure term was eliminating by taking curl twice of Eq. (10). Once again taking the curl of the resulting equation, the resulting equation and the Eq. (11) – (12) are then non-dimensionalized using the following transformations,

$$(x',y',z')=(x^*,y^*,z^*)d,\ t'=t^*\left(\frac{\gamma d^2}{\kappa_T}\right),\ \lambda_1=\frac{d^2}{\kappa_T}\lambda_1^*,$$
$$(u,v,w)=(u^*,v^*,w^*)\left(\frac{\kappa_T}{d}\right),\ \ T'=(\Delta T)T^*,\ S'=(\Delta S)S^* \tag{14}$$

The non dimensional equations (on dropping the asterisks for simplicity) are obtained as

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{V_a}\frac{\partial q}{\partial t}+\sqrt{T_a}\hat{k}\times q\right)q_a+\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\nabla p+Ra_T T\ \hat{k}-\hat{k}Ra_S S\right)=0 \tag{15}$$

$$\left[\frac{\partial}{\partial t}-\nabla^2+q.\nabla\right]T-w=D_f\frac{Ra_S}{Ra_T}\nabla^2 S \tag{16}$$

$$\left[\lambda\frac{\partial}{\partial t}-\nabla^2\frac{1}{L_e}\right]S+(q.\nabla)S-w=S_r\frac{Ra_T}{Ra_S}\nabla^2 T \tag{17}$$

Where $T_a=\left(\frac{2\Omega\kappa_T}{\nu\varepsilon}\right)^2$ is Taylor number, $V_a=\frac{\varepsilon\gamma\nu d^2}{K_Z\kappa_T}$ is Vadasz number, $Ra_T=\frac{\beta_T g\Delta T K_Z d}{\nu\kappa_T}$ is the thermal Rayleigh number, $Ra_S=\frac{\beta_S g\Delta S K_Z d}{\nu\kappa_T}$ is the solute Rayleigh number, $\lambda_1=(\frac{K_{11}}{\gamma d^2})\overline{\lambda_1}$ is relaxation parameter, $\lambda_2=(\frac{K_{11}}{\gamma d^2})\overline{\lambda_2}$ is retardation parameter, $L_e=\frac{\kappa_T}{\kappa_S}$ is Lewis number, $S_r=\frac{K_{ST}\beta_S}{\kappa_T\beta_T}$ the soret parameter, $D_f=\frac{K_{TS}\beta_T}{\kappa_T\beta_S}$ Dufour parameter , $\lambda=\frac{\varepsilon}{\gamma}$ normalized parameter, $\xi=\frac{K_x}{K_Z}$ is mechanical anisotropic parameter, $q_a=\left(\frac{1}{\xi}u,\ \frac{1}{\xi}v,\ w\right)$ is anisotropic modified velocity vector.

$$\frac{1}{\xi}\omega+\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{V_a}\frac{\partial\omega}{\partial t}-\sqrt{T_a}\frac{\partial\omega}{\partial z}\right)=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[Ra_T\left(\frac{\partial T}{\partial y}i-\frac{\partial T}{\partial x}j\right)-\frac{1}{L_e}Ra_S\left(\frac{\partial S}{\partial y}i-\frac{\partial S}{\partial x}j\right)\right] \tag{18}$$

where $\omega=\nabla\times q$ is vorticity vector. On further taking curl, we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{V_a}\frac{\partial}{\partial t}(\nabla^2 q)+\sqrt{T_a}\frac{\partial\omega}{\partial z}\right)-\mathrm{Q}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(Ra_T\nabla_1{}^2T-\frac{1}{L_e}Ra_S\nabla_1{}^2S\right) \tag{19}$$

where $\mathrm{Q}=(\mathrm{Q}_1,\mathrm{Q}_2,\mathrm{Q}_3)$,

$$\mathrm{Q}_1=\frac{1}{\xi}\frac{\partial^2 v}{\partial y\partial x}+\frac{\partial^2 w}{\partial x\partial z}-\left(\frac{\partial^2 v}{\partial y^2}+\frac{1}{\xi}\frac{\partial^2 u}{\partial z^2}\right),$$

$$\mathrm{Q}_2=\frac{1}{\xi}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}-\frac{1}{\xi}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right),$$

$$\mathrm{Q}_3=-\left(\nabla_1{}^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w$$

The above system will be solved by considering stress free and isothermal boundary conditions as given below:

$$w=\frac{\partial^2 w}{\partial z^2}=T=S=0 \ \text{ at } \ z=0, z=1. \tag{20}$$

Linear Stability Analysis

In order to study linear stability, the eigenvalue problem defined by Eq. (18) - (19) is solved. Taking vertical component the equation are

$$\omega_z=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\xi\sqrt{T_a}\frac{\partial w}{\partial z} \tag{21}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial \omega_z}{\partial z}+\left(\nabla_1^{\,2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(Ra_T\nabla_1^{\,2}T-\frac{1}{L_e}Ra_S\nabla_1^{\,2}S\right) \tag{22}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)^2 T_a\frac{\partial^2 \omega}{\partial z^2}+\left(\nabla_1^{\,2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(Ra_T\nabla_1^{\,2}T-\frac{1}{L_e}Ra_S\nabla_1^{\,2}S\right) \tag{23}$$

Where $\omega_z=\nabla\times w$. For liner stability analysis, normal mode technique is used to solve the Eigen value problem defined by Eq. (23), (16) and (17), by using time periodic disturbance in horizontal plane. For fundamental mode

$$\begin{pmatrix} w\\ T\\ S\end{pmatrix}=\begin{pmatrix} w_0\\ \Theta_0\\ \phi_0\end{pmatrix} exp\left(i\left(lx+my\right)+\sigma t\right)+\mathrm{Sin}\left(\pi z\right), \tag{24}$$

Where l, m are the horizontal wave numbers, $a^2=l^2+m^2$ and $\sigma=\sigma_r+i\sigma_j$ is growth rate. Substituting expression (24) into the lineralized eqs. (15) - (17), it is obtain

$$\left(\delta_1^2+\left(1+\lambda_1\ \sigma\right)^2\xi\ T_a\ \pi^2\right)w_0-\left(1+\lambda_2\ \sigma\right)a^2Ra_T\ \Theta_0+\frac{1}{L_e}\left(1+\lambda_2\ \sigma\right)a^2Ra_S\ \phi_0=0 \tag{25}$$

$$\left[\sigma+\delta^2\right]\ \Theta_0-w_0+\ \delta^2D_f\frac{Ra_S}{Ra_T}\phi_0=0 \tag{26}$$

$$\left[\lambda\sigma+\frac{\delta^2}{L_e}\right]\phi_0-w_0+\delta^2S_r\frac{Ra_T}{Ra_S}\Theta_0=0. \tag{27}$$

Where $\delta^2=\pi^2+a^2$, $\delta_1^2=\frac{1}{\xi}\pi^2+a^2$. The above equation can be expressed in matrix form as

$$\begin{pmatrix} \left(\delta_1^2+\left(1+\lambda_1\ \sigma\right)^2\xi\ T_a\ \pi^2\right) & -\left(1+\lambda_1\ \sigma\right)a^2Ra_T & \frac{1}{L_e}\left(1+\lambda_1\ \sigma\right)a^2Ra_S\\ -1 & \left(\sigma+\delta^2\right) & \delta^2D_f\frac{Ra_S}{Ra_T}\\ -1 & S_r\delta^2\frac{Ra_T}{Ra_S} & \lambda\sigma+\frac{\delta^2}{Le}\end{pmatrix}\begin{pmatrix} w_0\\ \Theta_0\\ \Phi_0\end{pmatrix}=0.$$

For non zero solution, the determinant of the matrix has to be zero, therefore, the thermal Rayleigh number can be obtained as

$$Ra_T=\frac{L_e}{a^2\left(1+\lambda_1\ \sigma\right)\left(L_e\lambda\sigma+\delta^2+\delta^2S_r\right)}\begin{pmatrix} \left(\delta_1^2+\left(1+\lambda_1\ \sigma\right)^2\xi\ T_a\ \pi^2\right)\left(\left(\sigma+\delta^2\right)\left(\varepsilon_n\sigma+\frac{\delta^2}{L_e}\right)-\delta^4D_fS_r\right)\\ +a^2Ra_S\left(1+\lambda_1\ \sigma\right)\left(\delta^2D_f+\frac{1}{L_e}\left(\sigma+\delta^2\right)\right)\end{pmatrix} \tag{28}$$

The growth rate σ is in general a complex quantity such that $\sigma=\sigma_r+i\ \sigma_i$. The system with $\sigma_r<0$ is always stable, while for $\sigma_r>0$ it will become unstable.

For neutral stability state $\sigma_r=0$

Stationary State

The steady onset corresponds to $\sigma=0$ (i.e. $\sigma_r=\sigma_j=0$) and becomes the Eq. (28). The value of the thermal Rayleigh number of the system for a stationary mode of convection is as given below:

$$Ra_T^{st} = \frac{1}{a^2\left(\delta^2+\delta^2 S_r\right)}\begin{pmatrix}\left(\delta_1^2+\xi\, T_a\, \pi^2\right)\left(\delta^4-\delta^4 D_f S_r L_e\right)\\ +a^2 Ra_S\left(\delta^2 D_f L_e+\delta^2\right)\end{pmatrix} \tag{29}$$

Oscillatory State

We set $\sigma_r = 0$ and $\sigma_j \neq 0$ $(\text{i.e. } \sigma = i\sigma_j)$ in Eq. (28) and removing the complex quantities from the denominator, to obtain

$$Ra_T^{osc} = \Delta_1 + i\sigma_j \Delta_2 \,. \tag{30}$$

$$\Delta_1 = \frac{A\, d_1 + \sigma^2 B\, d_2}{a^2\left(A^2+\sigma^2 B^2\right)}$$

$$\Delta_2 = \frac{A\, d_2 - B\, d_1}{a^2\left(A^2+\sigma^2 B^2\right)}$$

Where, $a_1 = \delta_1^2 + \left(1-\lambda_1\, \sigma^2\right)\xi\, T_a\, \pi^2$,

$$a_2 = 2\xi\, T_a\, \pi^2, \qquad a_3 = \frac{\delta^4}{L_e} - \delta^4 S_r D_f - \lambda\sigma^2, \qquad a_4 = \frac{\delta^2}{L_e} + \lambda\delta^2,$$

$$a_5 = \delta^2 D_f + \frac{\delta^2}{L_e} - \frac{1}{L_e}\sigma^2\lambda_1, \qquad a_6 = \lambda_1\delta^2 D_f + \frac{\delta^2}{L_e}\lambda_1 + \frac{1}{L_e},$$

$$A = \frac{\delta^2 S_r}{L_e} + \frac{\delta^2}{L_e} - \sigma^2\lambda\, \lambda_1, \qquad B = \lambda + \lambda_1\left(\frac{\delta^2 S_r}{L_e} + \frac{\delta^2}{L_e}\right)$$

$$d_1 = a_1 a_3 - \sigma^2 a_2 a_4 + a_5 a^2 Ra_S, \quad d_2 = a_2 a_3 + a_1 a_4 + a_6 a^2 Ra_S$$

For oscillatory mode $\Delta_2 = 0$ and $\sigma_i \neq 0$. The Rayleigh number Ra_T^{osc} has to be real; therefore, the expression for oscillatory Rayleigh number is obtained as:

$$Ra_T^{osc} = \Delta_1 \tag{31}$$

Nonlinear Stability Analysis

In this section, nonlinear stability analysis has been done using minimal truncated Fourier series. For simplicity, only two dimensional rolls are considered, so that all the physical quantities do not dependent of y. Taking curl to eliminate pressure term from Eq. (1 b) , introducing the stream function ψ by using $u = \frac{\partial\psi}{\partial z}$, w= -$\frac{\partial\psi}{\partial x}$ and then for steady state assuming $\frac{\partial}{\partial t} = 0$, it is obtained

$$\left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2} + T_a\, \xi\frac{\partial^2}{\partial z^2}\right)\left(Ra_T\frac{\partial T}{\partial x} - \frac{1}{L_e}Ra_S\frac{\partial S}{\partial x}\right) = 0 \tag{32}$$

$$\nabla^2 T + D_f\frac{Ra_S}{Ra_T}\nabla^2 S + \frac{\partial(\psi, T)}{\partial(x,z)} - \frac{\partial\psi}{\partial x} = 0 \tag{33}$$

$$L_e^{-1}\nabla^2 S + S_r\frac{Ra_T}{Ra_S}\nabla^2 T + \frac{\partial(\psi, S)}{\partial(x,z)} - \frac{\partial\psi}{\partial x} = 0 \tag{34}$$

It is to be noted that the effect of nonlinearity is to distort the temperature concentration fields through the interaction of ψ and T ,ψ and S As a result a component of the form $Sin(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi = A_1(t) sin(ax) sin(\pi z), \tag{35}$$

$$T = B_1(t)cos(ax)sin(\pi z) + B_2(t)sin(2\pi z), \tag{36}$$

$$S = C_1(t)cos(ax)sin(\pi z) + C_2(t)sin(2\pi z), \tag{37}$$

Where the amplitudes $A_1(t), B_1(t), B_2(t), C_1(t), C_2(t)$ are functions of time and are to be determined.

Steady Finite Amplitude Motions

Substituting above expressions (35)-(37) in Eqs. (32) - (34) and equating the like terms, the following set of nonlinear autonomous differential equations were obtained

$$(\delta_1^2 + T_a \ \xi \ \pi^2)A_1 + aRa_T B_1 - \frac{1}{L_e} aRa_S C_1 = 0 \tag{38}$$

$$aA_1 + \delta^2 B_1 + \pi a A_1 B_2 + \delta^2 D_f \frac{Ra_S}{Ra_T} C_1 = 0 \tag{39}$$

$$\pi a \ A_1 B_1 - 8\pi^2 B_2 - 8\pi^2 D_f \frac{Ra_S}{Ra_T} C_2 = 0 \tag{40}$$

$$aA_1 + L_e^{-1}\delta^2 C_1 + \pi a A_1 C_2 + S_r \delta^2 \frac{Ra_T}{Ra_S} B_1 = 0 \tag{41}$$

$$8\pi^2 L_e^{-1} C_2 - \pi a A_1 C_1 + 8\pi^2 S_r \frac{Ra_T}{Ra_S} B_2 = 0 \tag{42}$$

Numerical method is used to solve the above nonlinear differential equation to find the amplitudes. On solving for the amplitudes in terms of A_1, $B_1(t)$, $B_2(t)$, $C_1(t)$, $C_2(t)$ can be obtained.

Steady Heat and Mass Transports

In the study of this type problem, quantification of heat and mass transport is very important. The Nusselt number and Sherwood number denote the rate of heat and mass transports for the fluid phase. The Nusselt number and Sherwood number are defined by

$$Nu = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial T}{\partial z} dx}{\int_0^{\frac{2\pi}{a}} \frac{\partial T_b}{\partial z} dx} \right]_{z=0}$$

$$Sh = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial S}{\partial z} dx}{\int_0^{\frac{2\pi}{a}} \frac{\partial S_b}{\partial z} dx} \right]_{z=0} \tag{43}$$

Substituting the expressions of T, T_b, S and S_b in Esq. (43), it is obtained

$$Nu = (1 - 2\pi B_2) + D_f \frac{Ra_S}{Ra_T}(1 - 2\pi C_2) \tag{44}$$

$$Sh = (1 - 2\pi C_2) + S_r L_e \frac{Ra_T}{Ra_S}(1 - 2\pi B_2) \tag{45}$$

Further using the expressions of B_2, C_2 into Eq. (44, 45), the final the expressions of N_u and S_h can be obtained in terms of various parameters governing the system.

Results And Discussion

This paper investigates the effects of rotation and cross diffusion on stationary and oscillatory convection in a Maxwell fluid saturated anisotropic porous medium. The effects of various parameters such as mechanical anisotropy parameter, Solute Rayleigh number, Lewis number, relaxation parameter, Soret and Dufour parameters are computed and the results are depicted in figures. The neutral stability curves in the (Ra_T, a) plane for various parameter values are as shown in Figures. The values for the parameters are fined as $T_a = 100$, $\lambda_1 = .5$, $\lambda = .5$, $\mathrm{Ra}_S = 50$, $L_e = 5$, $S_r = .05$, $D_f = .05$ and $\xi = .5$, except the varying parameter. Figs.1 (a-f) is for stationary mode of convection, while Figs. 2 (a-g) are for oscillatory mode of convection.

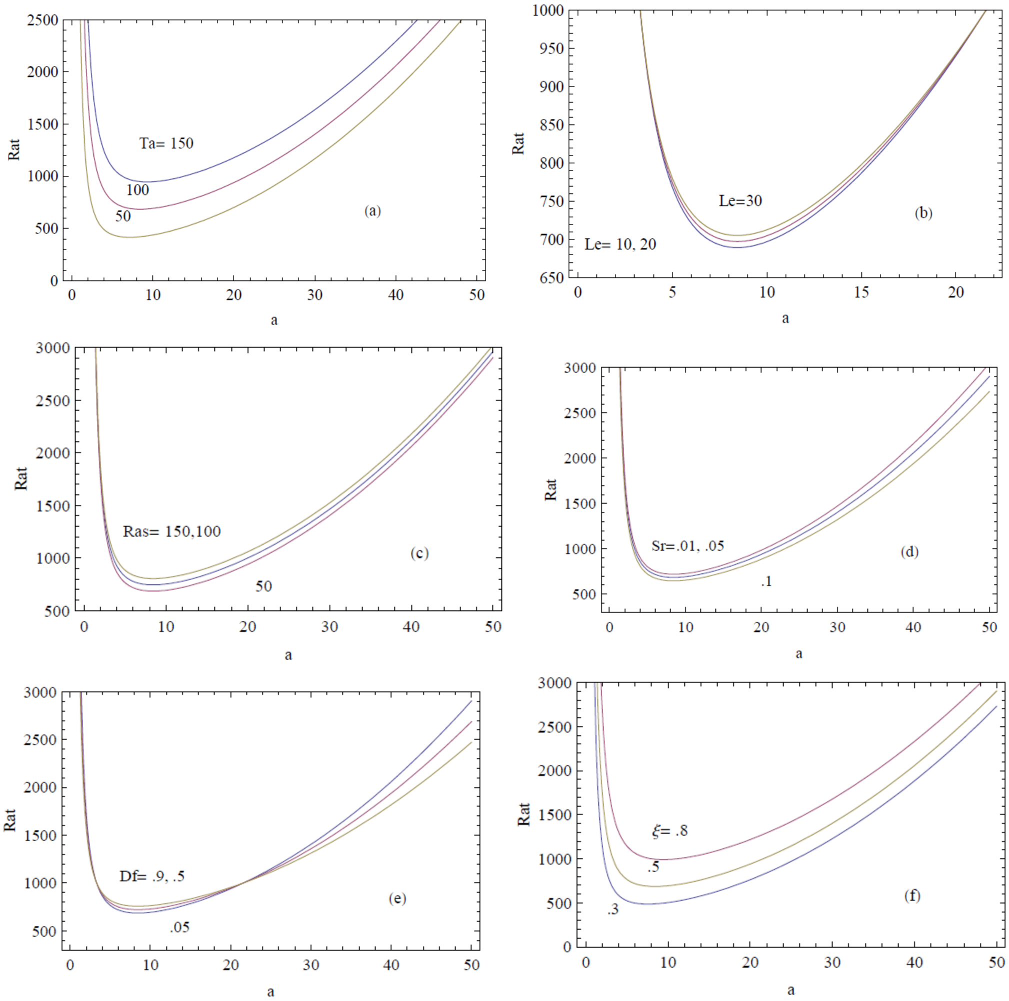


Figure1. *Stationary neutral stability curves for different values of (a) T_a (b) L_e (c) Ra_S (d) S_r (e) D_f (f) ξ*

In the Figs. 1 (a, c, f) and Figs. 2 (a, c, f), effects of the parameters; Taylor number T_a, solute Rayleigh number Ra_S and mechanical anisotropy parameter ξ are depicted respectively. From these figures, it is observed that an increment in the values of these parameters, increases the values of stationary and oscillatory Rayleigh number, thus stabilize the system, means onset of convection will take place at later point. In Figs. 1, 2 (b, e), it is shown that the effect of increasing Lewis number L_e and Dufour parameter D_f is to increase the value of Rayleigh number for stationary mode but decrease the value for oscillatory mode, thus to stabilize the stationary and destabilize the oscillatory mode of convection.

Further, Figs. 1, 2 (d), show the effect of Soret parameter, respectively for both stationary and oscillatory modes. It is found that increasing the value of the Soret parameter S_r decreases the value of stationary Rayleigh number, thus destabilizing the onset of stationary mode of convection but increases the value of oscillatory Rayleigh number, thus stabilizing the system. Also Fig. 2 (g) shows the effect of relaxation parameter on the onset of convection. It is observed that the oscillatory Rayleigh number decreases on increasing the value of the relaxation parameter λ_1, indicating that the effect of relaxation parameter is to destabilize the system, as the oscillatory convection takes place little early.

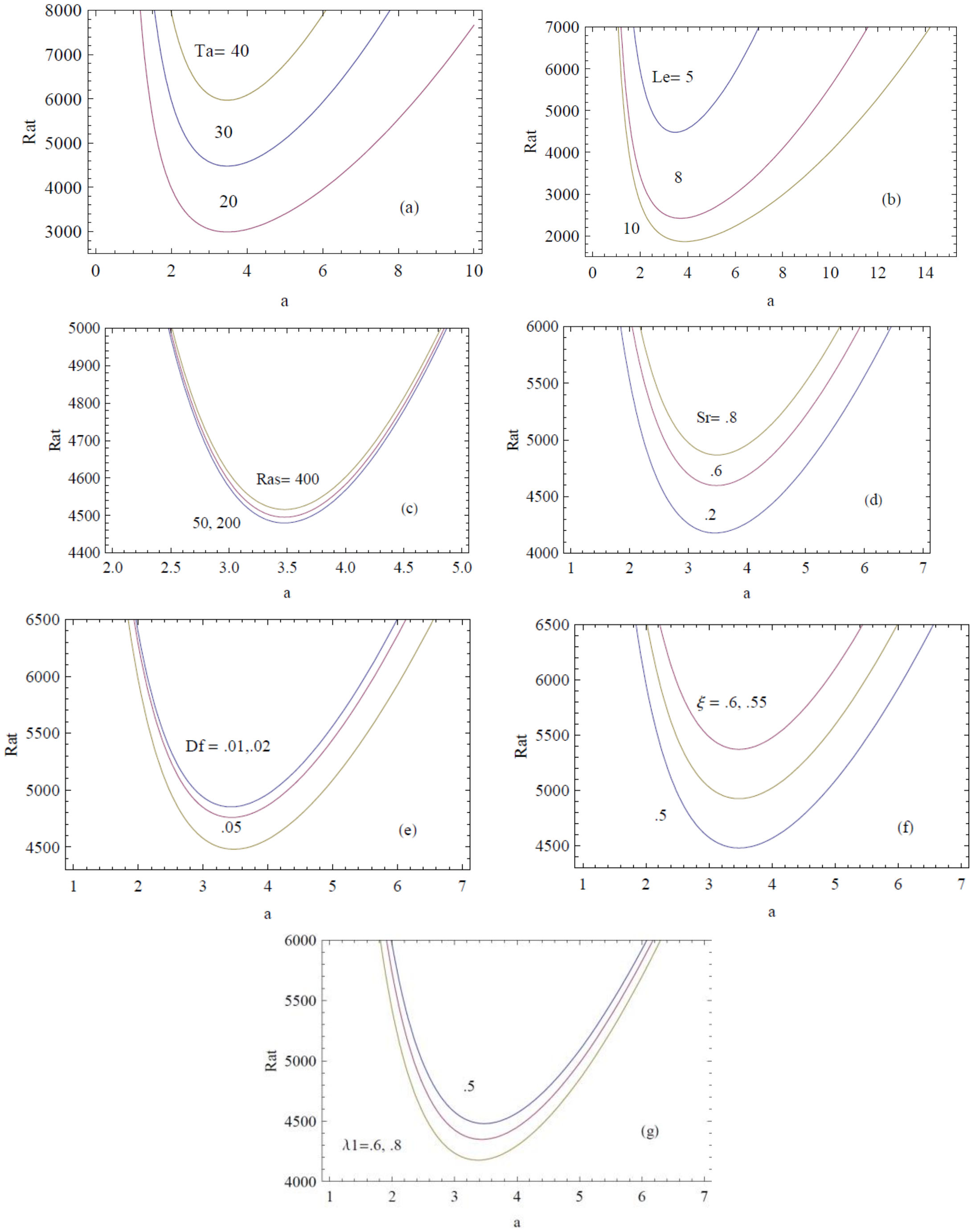


Figure2. *Oscillatory neutral stability curves for different values of (a)* T_a *(b)* L_e *(c)* Ra_S *(d)* S_r *(e)* D_f *(f)* ξ *(g)* λ_1

Now, the quantity of heat and mass transfer across the porous medium is computed in terms of the thermal Nusselt number and Sherwood number as functions of various parameters, which are fixed at T_a = 10, Ra_S = 50, L_e = 5, S_r =.05, D_f =.5 and ξ = .5 with varying one of the parameters. The results are depicted in the Figs.3 (a-f) through the Rayleigh-Nusselt number plane and in the Figs. 4 (a-f) through the Rayleigh-Sherwood number plane. It is found from the Figs. 3,4 (a, d),that the value of Nusselt number N_u decreases, while that of S_h increases on increasing the values of Taylor number T_a and of the Soret parameter S_r. This shows that the effect of Taylor number and Soret parameter is to decrease the heat transport, while increase the mass transport in the system. Figs. 3, 4(b) show that heat and mass transports increase on increasing the value of Lewis number L_e, thus destabilizing the system. It is found from the Figs. 3, 4 (c, e) that the effects of increasing Ra_S, D_f have a stabilizing effect on the system as heat transport increase and mass transport decrease on increasing the values of these parameters.

Further, it is found from the Figs. 3, 4 (f) that the effects of increasing ξ has a destabilizing effect on the system as heat and mass transport decrease on increasing the values of these parameters.

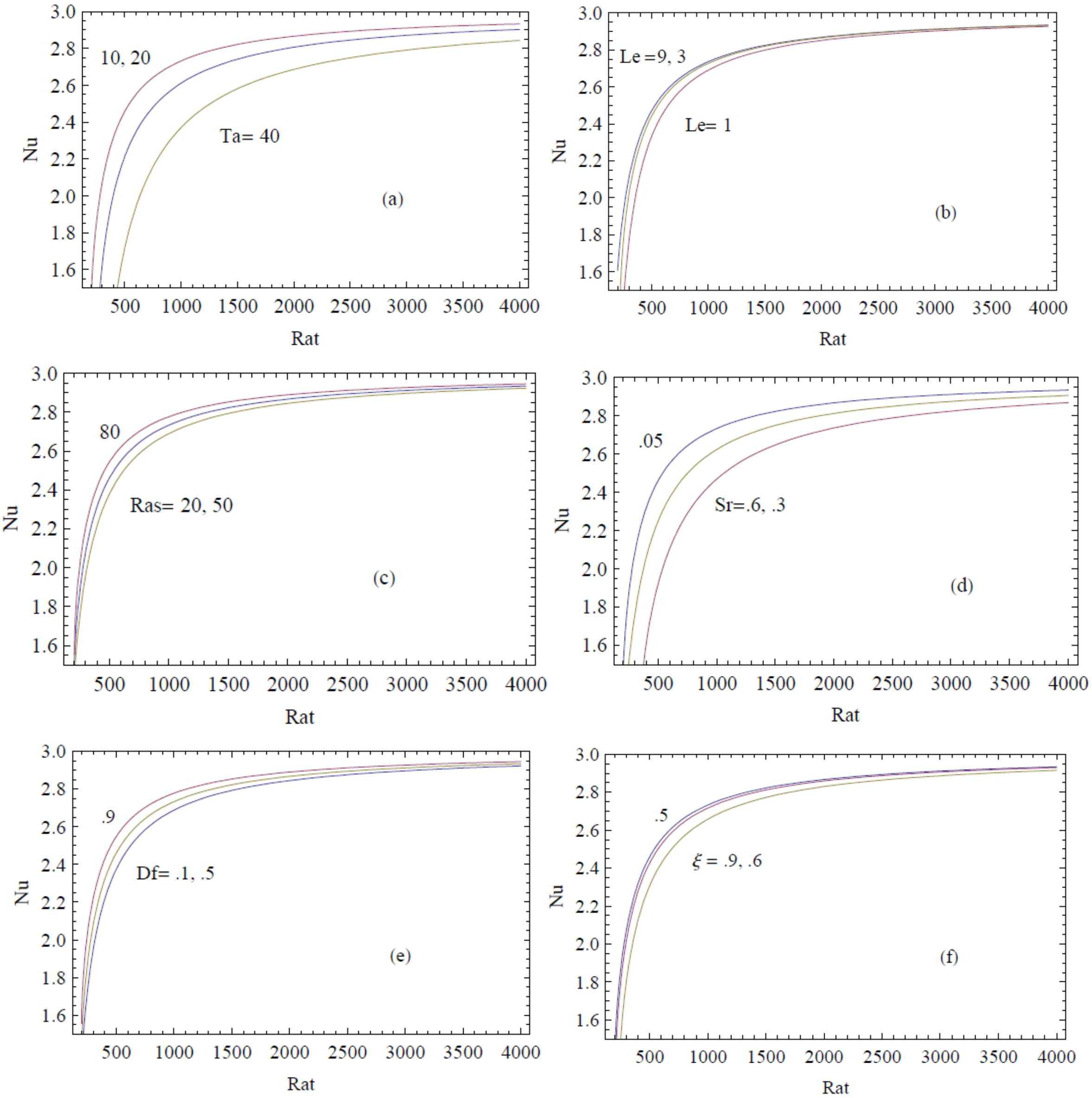
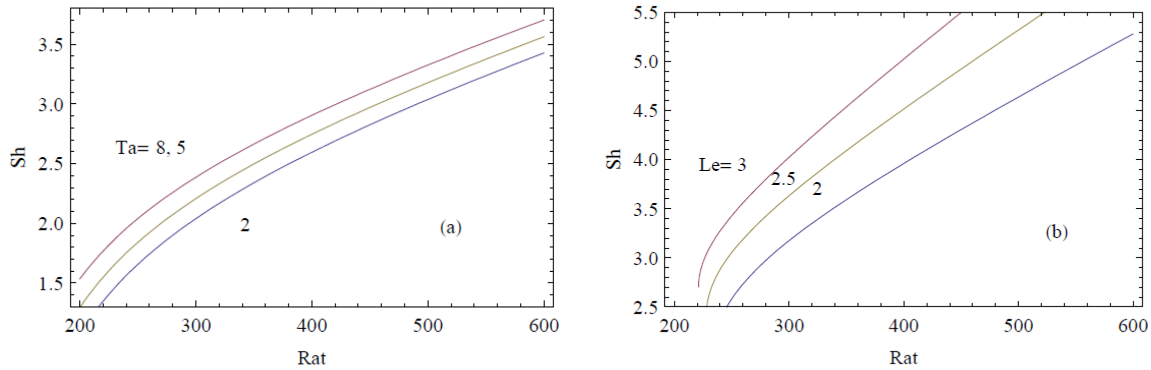


Fig.3 - *Nusselt number for different values of (a)* T_a *(b)* L_e *(c)* Ra_S *(d)* S_r *(e)* D_f *(f)* ξ



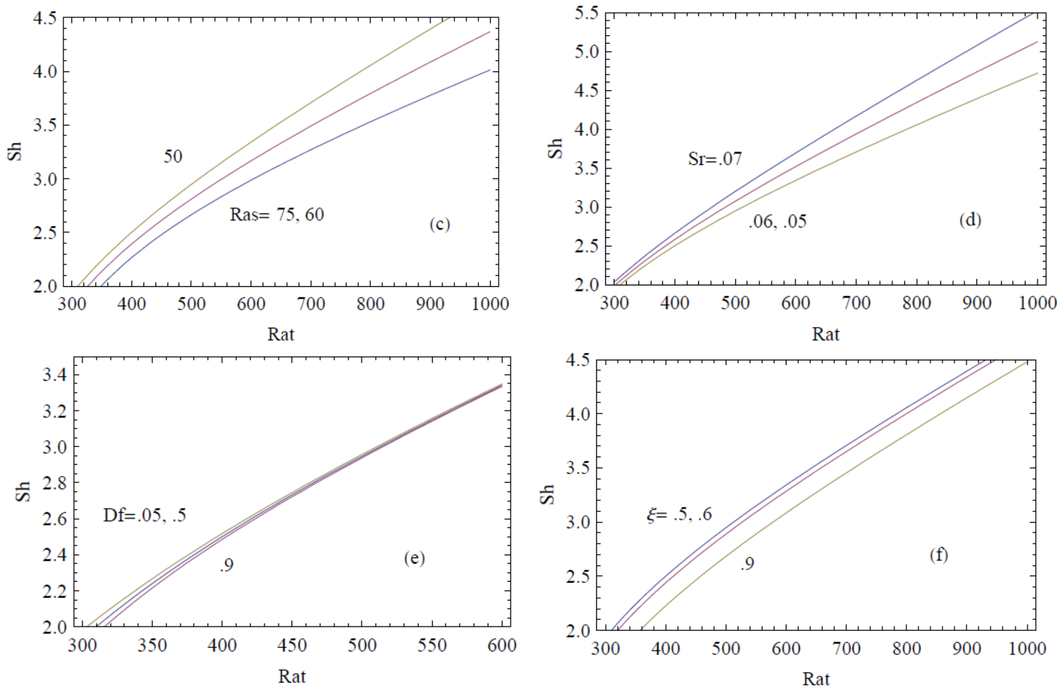


Figure4. *Graph between Sherwood number and Rayleigh number for different values of (a)* T_a *(b)* L_e *(c)* Ra_s *(d)* S_r *(e)* D_f *(f)* ξ

Conclusion

Soret effect and Dufour effect on cross diffusion convection in a rotating anisotropic porous medium saturated with a Maxwell fluid which is heated and salted from below, is investigated analytically using linear and nonlinear stability analyses.

The linear analysis is done using normal mode technique while the nonlinear analysis is based on a minimal representation of double Fourier series. Following conclusions are drawn:

- The Taylor number T_a, solute Rayleigh number Ra_S, and mechanical anisotropy parameter ξ have stabilizing effect on the system in both stationary and oscillatory modes of convection.
- The Soret parameter S_r has destabilizing effect on stationary mode while stabilizing effect on oscillatory mode of convection.
- The Lewis number L_e and Dufour parameter D_f have a stabilizing effect on stationary convection while opposite effect on oscillatory convection.
- The relaxation parameter λ_1 has destabilizing effect on oscillatory convection.
- The Lewis number L_e increases both Nusselt number and the Sherwood number.
- Increments in Solute Rayleigh number Ra_S and Dufour parameter D_f increase the Nusselt number and decrease the Sherwood number.
- Increments in mechanical anisotropy parameter ξ decrease both Nusselt number and the Sherwood number.
- Increments in Taylor number T_a and Soret parameter S_r decrease the Nusselt number and increase the Sherwood number.

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