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ABSTRACT

An attempt has been made in order to critically review the conceptual and mathematical modelling techniques as applied to a typical sandstone and fractured reservoirs. The typical as well as peculiar fluid dynamics associated with these reservoirs have been discussed in detail. It is emphasized in the present review that the focus on conceptual as well as mathematical models is far less for a petroleum reservoir than the focus associated with the numerical/analytical modelling. Developing conceptual and mathematical models associated with a fractured reservoir remains very challenging unlike a sandstone reservoir and the reasons associated with this complexity has been elaborated. The main problem stems from the fact that correlating the pore-scale capillary pressure with that of the macroscopic average reservoir pressure in a fractured reservoir remains challenging. In addition, using macroscopic Darcy's approach in order to conceptualize the microscopic capillary pressure requires further understanding as deducing a reasonable Representative Elementary Volume (REV) for a fractured reservoir at any scale is nearly impossible as fracture heterogeneities keep growing with scale. It is concluded that it requires a lot of understanding to delineate the dominant physical, chemical and biological processes associated with the fluid dynamics of a fractured reservoir; and developing the respective mathematical model for a fractured reservoir remains still more complex as non-linear hyperbolic nature of differential equations also come into picture in addition to the parabolic nature of differential equations.

Keywords: Conceptual model, mathematical model, fractured reservoir, microscopic-scale, macroscopic-scale.

INTRODUCTION

Reservoir simulation requires a very strong background on geological and geophysical modeling aspects as well, on top of a sound understanding on reservoir engineering and reservoir simulation techniques in order to better characterize the petroleum reservoir's "dynamic equilibrium". A fundamental distinction on various scales at which the problem of interest is focused at and the various scales associated with different data secured matters a lot, while deducing the appropriate mathematical model. A very strong conceptualization with only a limited core/laboratory/field data makes the reservoir modeller to deduce a very challenging mathematical model. Numerical modelling is all about solving the given set of coupled nonlinear partial differential equations. So, the fundamental questions that are very critical are

as follows: As a reservoir modeller, (1) How best will we be able to conceptualize a given reservoir?; and (2) How good we are at in translating the conceptual model into an equivalent mathematics?.

The queries such as those posed above, become very critical as describing flow through a porous medium is not as simple as described by the familiar Navier-Stokes Equation (NSE), where the various forces acting on the fluid mass is equated with the rate at which the fluid element gets accelerated. Even in the absence of inertial and turbulent effects, the fluid flow through a porous medium cannot be approximated by NSE, because, unlike flow through pipes, flow through a porous medium has two fundamental distinct entities namely solid-grains and porespaces. Both the entities are not of interest to analyse the fluid flow, while the fluid flow is

focused only through these pore spaces (or pore volumes). However, these solid-grains, despite not being useful for the fluid flow to occur, become an inevitable part of the porous medium as it is these solid-grains, which are going to decide the fundamental reservoir properties namely porosity and permeability in the case of single-phase fluid flow; and in addition, it indirectly decides the magnitude of capillary pressure and the respective fluid saturations of interest in the case of multi-phase fluid flow. Thus, the fluid flow analysis is carried out by considering the effective cross sectional area only by ignoring the spaces occupied by the solid grains, in a given cross sectional area. Thus, the presence of solid grains inhibits the feasibility of analysing the fluid flow through a porous medium at the microscopic scale. In other words, if the dependent variables of interest such as pressure and saturation are looked at the microscopic-scale, then, it will not be feasible for us to make use of the existing differential calculus as it requires a fundamental assumption that the spatial and temporal distributions of the dependent variables of interest should have smooth and continuous variation. This particular aspect forces us to look at the problem of analysing fluid flow through a porous medium at the macroscopicscale as against the conventional microscopicscale. The concept of Representative Elementary Volume (REV) is very fundamental associated with any continuum scale. For a sandstone reservoir, the concept of REV works well at the macroscopic-scale, while for a fractured carbonate reservoir, deducing a meaningful REV itself becomes very challenging as the degree of heterogeneity keeps increasing with the scale of the problem; and eventually, the parameter of interest like density does not reach an asymptoticity, irrespective of the increased volume. On the other hand, treating the fractured carbonate reservoirs as an Equivalent Porous Medium (EPM) will not yield good results as the fundamental relation between the volumetric flow rate and pressure gradient will not be linear in most of the cases, except with a fractured reservoir with abnormally high fracture density. In addition, Darcy's law needs to be used with utmost care as it describes that the direction of fluid flow is with decreasing pressure gradient. However, this concept alone is not sufficient to secure the details of the direction of fluid flow; and the concept of Hubbert's fluid potential needs to be applied in

order to secure the knowledge on the total hydraulic forces, which is the summation of the gravitational potential and pressure potential.

Since, Darcy's law is based on the continuumscale, looking at the problem at the pore-scale, where the concept of capillary pressure becomes very critical, becomes erroneous, Thus. extending the single-phase fluid flow Darcy's law to describe multi-phase fluid flow becomes very complex as the fundamental concept of permeability dependency only on reservoir property is violated, when Darcy's law is extended for describing multi-fluid flow through a porous medium. Apart from permeability becoming a function of both rock and fluid properties for describing multi-phase fluid flow, the concept of capillary pressure, which is the difference between the wetting and non-wetting phase is looked at the pore-scale, which again violates Darcy's assumption. As a matter of fact, this is the precise reason why laboratory-scale capillary pressure is required to get the fieldscale capillary pressure; and this capillary pressure cannot be measured in the field directly; and requires the use of Leverett J-Function for conversion. Thus, the conceptualization of fractured reservoir itself requires a serious attention unlike a sandstone reservoir.

Conceptual modelling remains the fundamental aspect for all modelling processes. In the recent past, more emphasis is being given to either analytical or numerical modelling, while the focus on the development of an improved mathematical model, conceived from the improved conceptual modelling seems disappearing. This is predominantly because of the two main aspects: (a) In petroleum engineering, most of the graduate students get placed comfortably in a reasonably reputed petroleum industry, and thereby, the number of graduate students opting for higher studies towards research becomes insignificant; and subsequently, the number of researchers who can really contribute to the science of petroleum engineering is far less in comparison with most of the other engineering disciplines; (2) The petroleum industry, on the other hand remains highly confidential in letting know their technology development associated with the heavy industrial competition; (3)Academicians from petroleum engineering mostly carry out their research work at the laboratory-scale (for example, core flooding), where the concept such as incorporating the real field-scale heterogeneity remains nearly missing; (4) Most of the

individuals associated with the petroleum engineering are convinced with using the already existing petroleum software packages without appreciating the limitations of their conceptual, mathematical and numerical modelling techniques; (5) many of the petroleum scientists hardly differentiate the importance of conducting fundamental research and applied research as the petroleum industry is mainly job-oriented and any funding towards projects require an immediate solution rather than focusing the problem from the fundamental aspects. For example, the petroleum industry still uses the Darcy's empirical relation (the momentum conservation equation used to describe fluid flow through a porous medium) deduced in 1856 in order to characterize the fluid flow through a saturated subsurface geological unit and as on date, there is no improved relation or classical laws to characterize the reservoir at the microscopic-scale. On the other hand, the concept of "nano" is widely used for characterizing the hydrocarbon reservoir. The concepts such as "nano-Darcy in case of tight reservoirs" and "nano-particles in case of chemical EOR" along with the usage of conventional macroscopic Darcy's law becomes highly questionable. Because "nano-scale" and "macro-scopic scale" are the kind of extremes for a petroleum reservoir; and merging both the scales in the same mathematical model (partial differential equation) seems incorrect. In other words, physically, it may not be correct to have both microscopic (for example, capillary pressure and contact angle) and macroscopic parameters (mean porosity, mean permeability and mean reservoir pressure) appearing in the same equation, although dimensionally the equation remains valid. Hence, a proper scaling / connectivity needs to be established in order to have a relation between microscopic properties with that of the macroscopic reservoir properties. Even then, bridging the gap between the different reservoir parameters is quite difficult. For example, the reservoir porosity is normally distributed in a field, while the reservoir permeability is log-normally distributed in the same field. Thus, there are issues even within the parameters of macroscopic scale itself. Thus, scaling is a very fundamental issue that deserves an enhanced attention particularly in the context of enhanced oil recovery. Different scales are being used by different people working on the same problem. Now, the issue is (a) the petroleum industry deals with the larger macroscopic fieldscale problems; (b) experimentalists, on the

other hand, mostly focus on the laboratory-scale investigations; and (c) very few academicians focus on pilot-scale problems. Given this background, who is really focusing the reservoir aspects at the microscopic-scale? It can be noted that we do have enormous static data available at the microscopic-scale with the technology development, while we do not have a meaningful mechanism to convert these static data for understanding the transient reservoir dynamics. And, this is where the concept of fundamental or basic research plays a crucial role, which would possibly help to connect all kinds of data available at varying scales can be clubbed together to better understand the complicated reservoir dynamics. Thus, a petroleum reservoir can better be characterized only when the concepts at the microscopic-scale (such as capillary pressure and wettability) is well understood along with the details available at the laboratory, pilot and field-scales. It can be noted here that all the laboratory scale investigations do not really reflect the fieldscale scenario, and we only have the ways to convert the laboratory-scale details into an equivalent field-scale value (for example, capillary pressure using Leverett J-Function). On top it, due to the limited industrial-academic interactions, characterizing a petroleum reservoir is becoming a real challenge.

CONCEPTUAL MODEL FOR A SANDSTONE RESERVOIR

The conceptual description of the physical/ chemical/biological processes encountered in a typical petroleum reservoir is highly complicated because of (a) the irregular and non-continuous multi-phase fluid flow paths reflecting the heterogeneity of the concerned geological unit associated with the pay-zone thickness; (b) the proximity of the solid grain boundaries to the mobile multi-phase fluids encountering a complex combination of viscous/capillary/ gravity dominated fluid flow - as a function of both media property (mean grain size, and in turn, intrinsic permeability) and fluid properties (fluid pressure and temperature); and (c) the mutual interactions of the multi-phase fluids within the pore-volume; the dissolved particulate matters associated with the miscible fluids within the pore-volume; and the solid grains (porous medium) – as a function of pressure and temperature. Given this back ground, it is not practically feasible to list the physical, chemical and biological processes of interest as a separate list.

Also, it is not possible to infer the interactions between the variables/parameters of interest physical/chemical/biological arising from processes. Then, we may be required to decipher whether the variables of interest are interacting among them linearly or non-linearly. If it is a non-linear interaction, then, the kind/type of non-linear (weak non-linear / strong non-linear) interaction needs to be understood; and finally, we should be able to delineate the dominant processes on physical/chemical/biological interest in order to ignore the rest of insensitive variables. Having deduced the parameters/ variables of interest, the next task is to deduce a reasonable reservoir geometry with the best possible well pattern scenarios. We also need to finalize the concerned co-ordinate system (Cartesian/Radial) with appropriate initial and boundary conditions. Thus, the final list containing (i) the dominant dependent and independent variables and parameters (constant coefficients and variable coefficients) of interest; (ii) the reservoir geometry with various well patterns; and (iii) appropriate initial and boundary conditions for the selected reservoir geometry; -all these things needs to be deduced at the cost of not using any modelling/ experimental/field studies. This is the kind of challenge one has to encounter in order to deliver a meaningful conceptual model of a typical petroleum reservoir, which eventually requires a lot of physical understanding (without any bias on any discipline) with the mathematical principles of the multi-phase fluid flow through a petroleum reservoir. At the end, one should be able to deduce a plot depicting a larger macroscopic field-scale reservoir system; along with a plot the sensitive physical/chemical/ depicting biological processes associated with the microscopic-scale so that one can easily understand the kind of complexity involved in bridging the gap between microscopic- and macroscopic-scale processes. Such a conceptual model would really reflect the field-scale petroleum reservoir to the best extent possible.

MATHEMATICAL MODEL FOR A SANDSTONE RESERVOIR

Regarding mathematical modelling, we still do not have an equation for describing fluid flow through a porous medium at the microscopicscale. Using Navier-Stokes analogy for describing fluid flow through a porous medium at the microscopic scale becomes questionable as the fluid flow through pipes/capillary-tubes has 100% porosity. Thus, it may not be correct to apply a mathematical model for a porous medium (generally, with a porosity which is much lesser than 100%) using a mathematical model meant for 100% porosity. Any physical system with 100% porosity pertains to have an infinite hydraulic conductivity and such a physical system completely deviates from Darcy's fundamental assumption of having geological unit with a finite hydraulic conductivity. Thus, empirical (and not classical) Darcy's equation is the only momentum equation that is widely used to describe fluid flow through a porous medium at the larger macroscopic-scale. Thus, deducing a conceptual model at the microscopic-scale is just not possible using Darcy's approach and we need a slightly a larger scale for framing the conceptual model in order to get rid-off the microscopic model. By doing so, all the heterogeneities and/or processes at the microscopic scale become redundant. In other words, all the physical/chemical/biological processes of interest at the microscopic scale are assumed not to influence the resulting reservoir fluid dynamics the concerned reservoir. Mathematically speaking, all the noises/perturbations/fluctuations /oscillations associated with the microscopicscale are assumed to be turned off without any shocks or discontinuities so that a reasonable Representative Elementary Volume (REV) can be thought to exist in order to bring the physical domain of interest to be under single continuum and subsequently to make use of macroscopic Darcy's law. While formulating the mathematical model using the concept of single continuum based REV, one needs to carefully assess the concepts of using "capillary pressure" and "wettability" effects that are predominantly applicable only at the microscopic-scale. It can be noted that the fluid flow through a porous medium can generally be categorized to be under laminar flow with viscous dominance, when the mean pore size of the reservoir is greater than 10 microns. On the other hand, the fluid flow through a porous medium can be categorized to be under capillary effects, when the mean pore size of the reservoir is less than 1 micron. Thus, both Reynolds Number and Capillary Number helps to under the type of fluid flow associated with a particular petroleum reservoir. And, the flow will be characterized by a combination of both viscous and capillary forces, when the mean pore size of the reservoir falls in between 1 and 10 microns. If it is so, then, we must have clarity on the range of mean pore size of the reservoir below which, the fluid flow will predominantly be characterized by the capillary forces so that the macroscopic-scale

Darcy's law could be comfortably ruled out in a typical sandstone or a carbonate reservoir. On the other hand, if the mean pore size of the reservoir falls anywhere less than 10 microns, then, capillary forces will also be coming into picture in addition to the viscous forces while characterizing the reservoir fluid dynamics. In such cases, it becomes inevitable to delineate the range of mean pore size that roughly delineates the microscopic and macroscopic scales. Till date, there is no clear consensus on the above issue even if it is a typical homogeneous sand isotropic reservoir. There are few earlier studies with the deduction of REV for single-phase fluid flow, while such studies are remote for the cases with multi-phase fluid flow with sensitive reservoir thermodynamics.

In the context of mathematical modelling, handling the fundamental reservoir parameters namely porosity and permeability is of very critical. The question is whether these parameters can be considered as a constant or can they be treated to be varying as a function of either independent (space and time) or dependent (pressure/density and/or temperature) variables. The moment Darcy's law is used to characterize the fluid flow in a reservoir, the permeability should not vary as a function of either dependent or independent variables; and it should be treated as a constant (connecting the relation between the Darcy flux and the pressure gradient) as Darcy's law assumes that the basic reservoir parameters are associated with homogeneous and isotropic reservoir; and also, Darcy's law is applicable under steady-state fluid flow conditions, and hence, treating porosity or permeability as a function of time under Darcy's umbrella is questionable. In fact, Darcy used the term "hydraulic gradient" and "hydraulic conductivity" while deducing his original empirical relation in the context of experimental hydraulics. From Darcy's experimental work, hydraulic conductivity is a function of both fluid and reservoir properties for a single-phase fluid flow. However, in the name of extended Darcy's law for multi-phase fluid flow, the intrinsic permeability (as against hydraulic conductivity) is used, which depends both on fluid and rock properties. From Darcy's perspective, intrinsic permeability is purely a reservoir or rock property only, and it does not depend on fluid property; and it is associated only with the single-phase fluid flow. However, the same term (intrinsic permeability) becomes a function of fluid property as well in addition to the rock property by associating it with the saturated multi-phase fluid flow in a petroleum reservoir, which Darcy was not aware of.

The issue is permeability becoming a function of both rock and fluid property may be completely correct but not in the context of Darcy's approach as varying scales are merged together by doing it so. And this is where formulation of mathematical modelling using Darcy's approach needs its utmost care. It is to be noted that even in the case of single-phase fluid flow, we require the mean pore size (through which the fluid flows) in order to estimate the intrinsic permeability but we often use mean grain size (d10 / d50 / dmean) to estimate the permeability as it is relatively difficult to estimate the mean pore size directly. Thus, we already have an approximation while estimating the permeability value. In addition, the mean value of permeability is deduced from a relatively fewer sample values. Thus, one more approximation for permeability value results from using the limited number of data sets. In addition, it should be noted that the mean value of permeability should be deduced from geometric mean as against the arithmetic and harmonic means as the permeability in the field is characterized by log-normal distribution. Further, deducing a mean value of permeability from a combination of data sets deduced from liquid-permeability experiments / gas-permeability experiments / field-pumping tests again introduce some approximations. Thus, permeability itself is associated with so much approximation and such approximated value is used for estimating the Darcy flux as a function of pressure gradient. And, this approximated Darcy flux is used to estimate the production rate as a function of the reservoir geometry.

In the context of porosity, porosity is estimated directly from permeability values in the absence of the reasonable mean reservoir porosity data. Such correlation between porosity and permeability holds good for homogeneous and isotropic reservoirs and not for all the reservoirs. It should be noted that both porosity and permeability are relatively less in comparison with that of the groundwater aquifer. In other words, the existing porosity-permeability relations should be used with caution, when applied to a deep seated petroleum reservoir.

Another important aspect in the context of mathematical modelling is to ensure a linear relation between Darcy flux and pressure gradient, when using Darcy's law. This relation becomes critical as the depth of the reservoir becomes increasingly high, where the reservoir is associated with an enlarged compaction resulting from over-burden stress. In such cases, the Darcy flux may follow non-linearly with the pressure gradient and subsequently, Darcy's law needs to be handled with care.

It is also to be noted that such non-linear relation is quite possible in the vicinity of the production well as well, where the fluid flow may not remain purely horizontal, but rather as a set of curvi-linear lines. In other words, the fluid velocity in the vicinity of the production well becomes bifurcated into its horizontal and vertical components associated with a twodimensional fluid flow. Thus, application of one-dimensional Darcy's law in the vicinity of production well needs a caution.

In the context of Reynolds Number, it is generally categorized that the fluid flow is laminar when Reynolds Number is less than unity, while for the Reynolds Number between 1 and 10, the flow is assumed to undergo a transition state from laminar to turbulent; and the flow becomes fully turbulent when the Reynolds Number exceeds 10. Such a classification is associated with the homogeneous and isotropic reservoir, while a petroleum reservoir has inherent heterogeneities and hence, the ensuring the fluid flow to be laminar in a petroleum reservoir is critical. In addition, the laminar fluid flow in a petroleum reservoir should be free from inertial forces as well, whether it is weak or strong inertial forces (before becoming turbulent) for Darcy's law to be used. Because, the concepts of fingering, by-passing, channelling are all associated with the fluid flow pertaining to the laminar flow with possible inertial effects, where Darcy's law cannot be used.

Thus, the essence of mathematical modelling is to convert the conceptualized physics into an equivalent mathematics. And such conversion from (a) sparse/limited data sets; (b) various non-uniform data sets from laboratory, pilot and field-scale studies; and (c) various dominant processes associated with different scales; formulating a meaningful mathematical model for a reliable production forecast becomes a real challenge.

CONCEPTUAL MODEL FOR A FRACTURED/ CARBONATE RESERVOIR

The conceptualization of fluid flow through a fractured petroleum reservoir requires an all together a different perception from the conventional sandstone reservoirs. It is not easy and straight-forward to deduce the equivalent values for the parameters of interest from a fractured reservoir. First and foremost, fluid flow through fractured reservoirs are characterized by a new element called "fracture", which is essentially different from its associated reservoir-matrix. Flow through fracture can be conceptualized to be the fluid flow through pipes with 100% porosity at the scale of a single fracture. The concept of "fracture porosity" is different from the porosity of the single fracture. Fracture porosity stems from the fact that the total void spaces associated with fractures to that of the bulk reservoir-volume. The concept of flow through pipes is significantly different from the concept of fluid flow through a porous medium. Thus, any conceptual modeller should understand clearly that the fractured reservoir cannot be conceptualized as a conventional single-continuum model, where a reasonable REV can be deduced for the entire reservoir, but rather a multi-continuum model resulting from the extreme variations in porosity and permeability occurring over a very short span, particularly at the fracture-matrix interface. Mathematically, multi-continuum models implies the presence of shocks or discontinuities within the reservoir domain and for such domains, deducing a single REV for the entire reservoir is nearly impossible. And hence, we have models such as dual-porosity or triple porosity models in order to capture the local scale details, while models such as discrete fracture network focus on capturing the large-scale details of a petroleum reservoir.

Following points may be noted, while deducing a conceptual model for a fractured petroleum reservoir (Suresh Kumar and Ghassemi, 2005; Suresh Kumar and Sekhar, 2005; Suresh Kumar, 2008; Natarajan and Suresh Kumar, 2011; Renu and Suresh Kumar, 2012; Suresh Kumar, 2014; Renu and Suresh Kumar, 2014; Sivasankar and Suresh Kumar, 2014; Srinivasa Reddy and Suresh Kumar, 2015; Suresh Kumar, 2015; Suresh Kumar, 2016; Renu and Suresh Kumar, 2016; Vivek and Suresh Kumar, 2016; Sivasankar and Suresh Kumar, 2017; Renu and Suresh Kumar, 2017; Bagalkot et al., (2018); Natarajan and Suresh Kumar, 2018; Suresh

Kumar and Rakesh, 2018; Renu and Suresh Kumar, 2018a & 2018b):

- For sandstone reservoirs, it is straight-forward to apply the concept of hydrostatic environment in the absence of any internal movement of the reservoir fluids before puncturing the reservoir; and under such conditions, the associated maximum internal pressure gradient would be purely vertical resulting from the gravitational weight of the overlying fluids. Thus, it is clear that all the internal forces associated with the hydrostatics of a sandstone reservoir will be oriented vertically with buoyant force playing the major role. However, all the internal forces associated with the hydrostatics of a fractured reservoir need not necessarily be oriented vertically as the hydraulic gradient between any two fracture intersections; the degree of fracture connectivity: the distribution of fracture spacing, fracture width and fracture thickness; the intensity of irregular and disconnected fractures; and the fracture dip/orientation also play a critical role in addition to the density and gravity effects. Hydraulically speaking, the pore fluid pressure may not remain nearer to its hydrostatic conditions, when the fluids within the fractures of varying thicknesses continue to get adjusted. This is because the resultant internal pressure build up in a network of disconnected fractures will not be held at its minimum. Thus, the maximum rate of pressure increase need not necessarily be in the vertical direction for fractured reservoirs; and keeping this aspect during the conceptualization will help to deduce better conceptual models. Thus, in a fractured reservoir, before puncturing the reservoir associated with the static body of fluids, there could still be some force imbalances and the respective level of potential energy of the fluids may not remain a constant throughout. In other words, a fractured reservoir may have the characteristics of a hydrodynamic environment (with internal force imbalances) well ahead of puncturing subsequently, the reservoir; and the formation brine, oil and gas may not be following the same migration paths; and their respective final entrapment depends on both the fluid dynamics and the local-geology.
- The multi-phase fluid flow associated with a sandstone reservoir has a clear migration pattern of the fluids. The brine will be

moving in response to the flow force generated by the changes in their potential energies, while oil and gas will try to orient themselves along with the lines of their minimum potential energies. And, the resultant tilted oil-water and gas-water contacts would remain perpendicular to the orientation of the individual force vectors. However, in a fractured reservoir, the oil-water and gaswater contacts may not remain parallel to the iso-potential traces and these contacts would remain titled to the specific force vectors. i.e., they are tilted downwards in the direction of fluid flow. It can be noted that tilt is a function of the brine density; oil/gas density; and the slope of the potentiometric surface. In essence, the mechanism associated with the hydraulic segregation of water, oil and gas in a fractured reservoir would significantly deviate from that of a sandstone reservoir.

- In a sandstone reservoir, since the total volumetric pore space available is significant and well-connected, it is easy to observe the diminishing potential energy levels of the fluids along the direction of fluid migration, while the same observation would remain difficult due to the limited pore spaces available through fractures, which are not necessarily well-connected. In case, if the fluid flow through the fracture network is not hydraulically dynamic, then, it will be nearly impossible to define the movements of brine, oil and gas within a fractured reservoir.
- In case of a sandstone reservoir, the frictional resistance is encountered along the entire cross-section perpendicular to the direction of fluid flow. However, in a fractured reservoir, the mobile fluid is mostly associated with the high permeable fracture, and thus, the dominant frictional resistance is encountered only along the fracture walls. And also, the pressure gradient exits both along the fracture, i.e., along the dominant flow direction, and also, across the flow direction, i.e., between fracture and rockmatrix. Thus, in case of fractured reservoirs, it is not straight-forward to estimate the computation of net pressure force, which is a function of pressure-gradient, total cross sectional area and the porosity; and estimating these parameters become a difficult as the fractured reservoirs is associated with both mobile fluids as well as immobile fluids. It should be noted that

stored/trapped oil within the low-permeable rock-matrix pertains to the immobile fluids, while the extracted fluids from the rockmatrix into the fracture, and which is finally driven towards the production well pertain to the mobile fluids. Similarly, the computation of gravitational force becomes complex as finding the resultant density for the associated multi-phase fluids; and the resultant vertical gradient (sin theta) for the complex fracture network. In addition, in the context of resistive forces within a reservoir rock, the forces opposing the mobility of the fluids due to frictional drag is in general directly proportional to the Darcy flux, which is determined by the smooth distribution of pore fluid velocities; and the same is inversely proportional to the total area of fluid-solid contact. In case of sandstone reservoirs, the assumption of smooth distribution of pore fluid velocities is reasonable, while the pore fluid velocities in a fractured reservoir vary over several orders of magnitude between fracture and rockmatrix. Similarly, the area of the fluid-solid contact (specific surface area) in a sandstone reservoir increases proportionately with increasing scale/volume, while in a fractured reservoir, the fluid-solid contact area does not increase proportionately with increasing scale/volume. Thus, the estimation of both driving and resistive forces is not straightforward in a fractured reservoir.

- The concept of inertial energy might become very significant in case of a fractured reservoir associated with the fluid flow high permeable fractures. In such cases, using Darcian approach to describe the fluid flow as used in a sandstone reservoir may not be correct; and the fluid flow through fractured reservoir might require non-Darcian approach. Also, fractured reservoirs explicitly require a second order permeability tensor as the direction of reservoir fluids in a fractured reservoir depends on fracture orientation/ strike/dip. unlike the assumption of horizontal fluid flow associated with the sandstone reservoir. Subsequently, the concept of permeability in a fractured reservoir becomes highly space- and time-dependent.
- Fractured petroleum reservoirs are characterized by the presence of secondary and/or tertiary porosities (developed by a diagenetic process following the deposition) as against the conventional primary porosity associated

with a sandstone reservoir. The primary porosity that develops during the initial deposition of sediments (generally, in terms of millions of years ago) does not vary significantly from its initial value, at the time of the first drilling, while the porosity values associated with a fractured reservoir have a significant variation from its initial value, at the time of first drilling.

- Porosity values associated with a sandstone reservoir are normally distributed, while a fractured reservoir has essentially two different porosities, namely fracture porosity and matrix porosity. Porosity of a single fracture is 100%, while the magnitude of fracture porosity associated with a network of fractures is in general, an order of magnitude lesser than that of the porosity of a single fracture. Fracture porosity at the scale of a fracture network depends on the fracture density, fracture spacing, fracture width and fracture orientation/dip. The magnitude of matrix porosity generally varies between 1 and 10% and it depends essentially on the degree of compaction resulting from overburden stress.
- Any averaging technique used to deduce the mean porosity of the entire fractured reservoir computed as a function of individual fracture and matrix porosities may not be meaningful as both fracture and reservoir-matrix pertain to two different continuums as against the single-continuum concept associated with the sandstone reservoir.
- In case of sandstone reservoirs, the total hydrocarbon volume is associated with the total porosity, while the actual hydrocarbon volume that is recoverable is associated with effective (hydraulically connected) the porosity. It can be noted that there will not be significant difference between the total and effective porosities in case of sandstone reservoirs. However, in case of a fractured reservoir, the following porosities are to be taken into account: (a) porosity at the scale of a single fracture (100%); (b) fracture porosity (generally less than 10%; and; it depends on imbalances in tectonic forces; and physical and chemical weathering); (c) rock-matrix porosity (generally less than 5%; and it depend on the degree of rock compaction); (d) total porosity (generally falls between 1 and 15%; and it is a combination of fracture and matrix porosities); and (e) connected

porosity (generally falls between 25 to 75% of the total porosity).

- There is no physical correlation between fracture and matrix porosities. Fracture porosity results from the extent of physical and chemical weathering exposed by the reservoir rock, while matrix porosity is a function of degree of rock compaction.
- Permeability of a fractured reservoir essentially results from the fracture permeability, while the matrix permeability is in general several orders of magnitude lesser than that of the fracture permeability. Thus, the intrinsic permeability of a fractured reservoir essentially depends on the square of the mean fracture aperture thickness, while the intrinsic permeability of a sandstone reservoir depends on the square of the mean grain size that was developed at the time of rock formation.
- It may not be correct to estimate the permeability of a fractured reservoir using the statistics of fracture density, fracture spacing, fracture width and fracture dip as the "connectivity" of the resultant fracture network holds the key in deciding the actual permeability of the entire fractured reservoir. In this context, any complicated, developed 3-D fracture network model may not necessarily yield better results as the fluid is forced to flow through all the generated fracture network connections, while in reality, the fluid flow may not flow through all the developed fractures. This is because fluid flow through a fracture network not only depend on the fracture connectivity but also on the hydraulic gradients developed between each fracture intersection (which is a function of fracture dip/inclination).
- In case of sandstone reservoirs, permeability is estimated indirectly as a function of square of mean grain size, which generally varies very widely. However, in a fractured reservoir, the permeability of the fracture is estimated as a function of square of the fracture aperture thickness. Thus, it should be noted that the permeability is associated with grain-size for sandstone reservoirs, while the permeability estimation is associated with the pore-size itself.
- In case of sandstone reservoirs, the intrinsic permeability is generally assumed to be independent of time as per the Darcy's steady-state fluid flow assumption. However, in a fractured reservoir, the permeability is

directly related with the thickness of the fracture aperture, which is very sensitive to the rock deformation kinetics; poro-elasticity; and thermos-elasticity. Thus, assuming a constant value of permeability for fractured reservoirs would yield poor results.

- In case of sandstone reservoir, storage and transmission of hydrocarbon fluids take place simultaneously, while in a fractured reservoir, the storage and transmission of hydrocarbon fluids remain separated. Storage of hydrocarbon fluids is associated with the low-permeable reservoir-matrix, while the transmission of hydrocarbon fluids is associated with the high-permeable fractures.
- In a sandstone reservoir, it is quite logical to deduce a possible correlation between the results observed at the laboratory-scale (using core-flooding studies) to a larger fieldscale investigation to some extent. However, in a fractured reservoir, the experimental observations at the laboratory scale has very limited relevance in the context of extrapolating the same to a larger field-scale investigations. This is because, any understanding and its extrapolation to a larger field-scale, based on core-flooding studies is highly limited to reservoirs, where a well-defined REV exists. However, in case of fractured reservoirs, it is nearly impossible to deduce a reasonable REV as the heterogeneities associated with a fractured reservoir keeps increasing with scale/volume. For example, when microbes/ nutrients/alkaline/surfactant/polymer is injected into a core-flooding experiment, it should be clearly understood that the maximum sweeping (both vertical and aerial) of the injected solutes will be achieved with ease as the entire volume of the concerned core is extremely small with reference to the entire reservoir. On top of it, the kind of heterogeneities associated with the fractured reservoir can never be captured with a coreflooding study. And, the problem becomes even more serious when dealing with the investigations associated with EOR. Thus, the details associated with the microscopicdisplacement efficiency and macroscopic/ volumetric sweep efficiency from coreflooding experiments has almost no relevance, when the same gets extrapolated to a larger field-scale.
- In case of fractured reservoirs, understanding the vertical and horizontal resolution details becomes extremely important, during the

final interpretation associated with the reservoir production. As a good modeller, we should be clearly conceptualize the aerial extent from where the data have been gathered and the aerial extent with the missing data in the context of overlapping between horizontal and vertical resolution of a given reservoir geometry. And, this is where, the concept of mobility ratio and its associated vertical, horizontal, areal and volumetric sweep efficiencies are going to play a sensitive role; and in turn, a reservoir manager got to take a very sensible decision with reference to a given EOR project.

• It should be noted by a conceptual modeller that they have the least reservoir coverage from any core data, while the seismic data has the maximum reservoir coverage; and it is required to be understood, the details at the pore-scale that deals with the dynamics of wetting and non-wetting phases. So, the challenge for a conceptual modeller is as follows: (1) How best the seismic data can be down-scaled?; while (2) To what extent, the core data could be up-scaled? in order to better capture the pore-scale reservoir dynamics.

MATHEMATICAL MODEL FOR A FRACTURED /CARBONATE RESERVOIR

- Since storage and transmission of hydrocarbon fluids are associated with two different geological units namely reservoir-matrix and fracture, mathematical modelling of fluid flow through a fractured reservoir requires a multi-continuum as against the singlecontinuum concept associated with the sandstone reservoir.
- Unlike a sandstone reservoir, deducing a reasonable REV for a whole fractured reservoir remains highly challenging as the heterogeneity keeps increasing with the scale/volume of the fractured reservoir.
- In a sandstone reservoir, there is no physical/ chemical interaction between hydrocarbon fluids within the pore-space and the solid grains (ignoring the wettability). However, in a fractured reservoir, there is a definite interaction between the two different continua namely pore-space (fracture) and solid-grains (reservoir-matrix).
- In case of a sandstone reservoir, the no-slip boundary condition is not applied for every individual pore, while the same is applied along the solid/impermeable reservoir boundaries

as the entire geological unit is treated as a single continuum. However, in a fractured reservoir, since, fracture is a separate continuum, applying no-slip boundary conditions on the fracture walls remains challenging as fluid mass exchange takes place through this fracture wall.

- Even with a sandstone reservoir, characterizing the fluid flow regimes becomes difficult, when gas is also present in addition to the brine and oil as the compressibility of gas is significantly different from oil and water. However, in a fractured reservoir, the classification of flow regimes becomes further complex as the permeability of fracture is several orders of magnitude higher than that of the matrix permeability.
- The modelling associated with a sandstone reservoir involves a pore-geometry, whose characteristics lengths do not vary much, while, in case of a fractured reservoir, the pore geometry is essentially characterized by high permeable fractures. The characteristic lengths of such fractures vary non-uniformly unlike a sandstone reservoir. For example, the length of the fracture runs over tens and hundreds of meters; the width of the fracture runs over several tens and hundreds of centimetres, while the thickness of the fracture aperture runs over several tens and hundreds of microns. And, this is where, associating a fracture model with a particular scale becomes challenging.
- In case of sandstone reservoir, the geometry of the pore size (i.e., the mean grain size) does not significantly influence the flow regimes of the hydrocarbon fluids, while in a fractured reservoir, the size of the pore, i.e., the fracture aperture thickness is very sensitive in characterizing the flow regimes of the hydrocarbon fluids. For example, when the thickness of the fracture aperture exceeds 1000 microns, it is going to be mostly fluid flow through a network of connected fractures, while the interaction between fracture and matrix will be almost nil. Hence, for applications, where injection ofhot-

water/steam/alkaline/surfactant/polymer/nan o-particles/microbes is involved, and where the fluid/heat exchange has to happen from fracture into rock-matrix, the system will not at all be efficient, when the fracture aperture thickness exceeds 1000 microns. On the other hand, when the thickness of fracture

aperture varies in between 10 and 1000 microns, then, the above mentioned applications will have the maximum efficiency as in this range of fracture aperture thicknesses, the intensity of coupling between fracture and rock-matrix remains the maximum. However, when the fracture aperture thickness falls below 10 microns, then, more and more capillary forces try to dominate the show (as against the viscous and gravity forces) in driving the fluid flow and such fractured reservoirs again will not be vielding any reservoir efficiency. Thus, the variations in fracture aperture thickness have all together very different fundamental mechanisms associated with it in driving the fluid flow through a fractured reservoir.

- The intrinsic permeability varies over several orders of magnitude both in sandstone as well as in a fractured reservoir. But there is a huge difference between two distributions. In case of sandstone reservoirs, the variations occur over a very large scale/volume, while in case of a fractured reservoir, the permeability varies by several orders of magnitude just over a few microns, i.e., at the fracture-matrix interface, and this is where, a mathematical discontinuity develops at the fracture-matrix interface, which essentially violates the fundamental assumption of "smooth and continuous variation of variables of interest" associated with the basic calculus. And this is the reason why, a fractured reservoir cannot be treated to have a singlecontinuum and it requires a multi-continuum approach. This fluid/heat exchange term, also called the coupling term, encountered at the fracture-matrix interface is very sensitive to the selection of numerical solution techniques as ensuring the continuity of fluid/heat fluxes at that interface for the given mathematical model becomes a real challenge.
- The concept of pressure gradient is not easy in case of a fractured reservoir unlike a sandstone reservoir, where the fluid flow is driven by the direction of the single dominant pressure gradient. In case of fractured reservoirs, there is a primary pressure gradient acting along the high permeable fracture, while one more secondary pressure gradient acts that drives the fluid flow between the highpermeable fracture and low-permeable rockmatrix. For example, in thermal EOR, when steam is injected into the reservoir, the fluid first flows through the fracture resulting from

the primary pressure gradient along the fracture, while the same steam gets convected and/or conducted from fracture into the rock-matrix by the secondary pressure gradient between fracture and rock-matrix. Thus, deducing the resultant fluid flow direction associated with a fractured reservoir is not as easy as it is encountered in a sandstone reservoir.

In a sandstone reservoir, the mean fluid velocity of the reservoir fluids, which in turn, decide the residence time of the fluid mass within the porous reservoir does not create a significant impact in the context of thermal /microbial/chemical EOR. However, in the case of a fractured reservoir, the mean fluid velocity of the injected fluids (which may contain hot-water/steam/microbes and nutrients / alkaline/surfactant/polymer/nano-particles, depending on the concerned type of EOR) plays a crucial role. For example, in case of microbial EOR, the injected microbes and nutrients get transported along the fracture as a function of velocity of the mobile fluids. This fluid velocity determines the mean residence time of the solutes (microbes and nutrients in this case: alkaline /surfactant/ polymer/nano-particles in case of chemical EOR) associated with the injected fluids. This fluid velocity is a function of the square of the mean fracture aperture thickness. When this mean fracture aperture thickness falls in between 10 and 1000 microns the exchange of fluxes (fluid mass flux in case of chemical/microbial EOR: and heat flux in case of thermal EOR) at the fracture-matrix interface will be very efficient. In addition, with this range of fracture aperture thickness if velocity of the reservoir fluids also remain relatively lesser, then, the solutes get the maximum residence time of solutes within the fracture to traverse from one fracture junction to the other. Enhanced residence for solutes within the fracture implies an increased exchange of fluid-mass/heat from the fracture into the rock-matrix. When more solutes are transferred through the fracturematrix interface. more bio-surfactants/ chemical-surfactants will be produced or more heat will be transferred; and in turn, the interfacial reduction between stored/trapped oil and water will be very efficient, which will subsequently increase the mobility of the stored/trapped oil within the low-permeable rock-matrix. On the other hand, when the

fracture aperture thickness is larger than 1000 microns and accompanied by a higher velocity of injected fluid-mass/hot-water/ steam, then, there can be a very limited exchange of injected fluid-mass/heat flux from fracture into the rock-matrix; and in turn, the chemical-surfactant/ bio-surfactant production or net heat transfer through the fracture-matrix interface will be very poor. Subsequently, the mobility of the stored/ trapped oil within the rock-matrix will be very less and the reservoir efficiency will not be significant. Thus, the residence time of the injected fluids within the high permeable fracture plays a crucial role in deciding the reservoir efficiency in the context of thermal/ microbial/chemical EOR. It should be noted that none of the existing petroleum software packages takes care of the concept of residence time of the injected fluids and its associated reservoir efficiency in the context of EOR.

In case of a sandstone reservoir, the physical system tries to reach the steady-state flow condition after its initial perturbations (in terms of initial heterogeneities); and hence, conventional parabolic dominant the diffusivity equation can be comfortably used to describe the fluid through a porous sandstone reservoir. However, in a fractured reservoir, the initial heterogeneities do not try to vanish, and rather, such heterogeneities keep growing with space and time, and hence, using the conventional parabolic dominant fluid flow equation using the concept of vanishing heterogeneities with larger time may not be applicable; and the nonlinear quadratic hyperbolic pressure gradient term needs to be retained along with the parabolic term in describing the fluid flow through a fractured reservoir. The hyperbolic term essentially conserves the property of interest without allowing the properties (heterogeneities) to get vanished with time.

CONCLUSION

The conceptual and mathematical modelling techniques as applied to a typical sandstone and fractured reservoirs has been critically reviewed. The typical as well as peculiar fluid dynamics associated with these reservoirs have been discussed in detail. It is emphasized in the present review that an enhanced focus is required for developing the conceptual as well as mathematical models associated with a fractured petroleum reservoir. Developing conceptual and mathematical models associated with a fractured reservoir still remains very challenging unlike a sandstone reservoir.

- A petroleum reservoir can better be characterized only when the concepts at the microscopic-scale (such as capillary pressure and wettability) is well understood along with the details available at the laboratory, pilot and field-scales.
- A meaningful conceptual model of a typical petroleum reservoir requires a lot of physical understanding (without any bias on any discipline) with the mathematical principles of the multi-phase fluid flow through a petroleum reservoir.
- The essence of mathematical modelling is to convert the conceptualized physics into an equivalent mathematics. And such conversion from (a) sparse/limited data sets; (b) various non-uniform data sets from laboratory, pilot and field-scale studies; and (c) various dominant processes associated with different scales; formulating a meaningful mathematical model for a reliable production forecast becomes a real challenge.
- It requires a lot of understanding to delineate the dominant physical, chemical and biological processes associated with the fluid dynamics of a fractured reservoir; and developing the respective mathematical model for a fractured reservoir remains still more complex as nonlinear hyperbolic nature of differential equations also come into picture in addition to the parabolic nature of differential equations.

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