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Comparison of LN2 and WB2 Probability Distributions for Low-flow Frequency Analysis

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ABSTRACT

Estimation of low-flow for a desired duration ('d' in days) and return period ('T' in years) is utmost importance for the assessment of water resources for many direct and indirect uses viz., municipal, irrigation, hydropower, public water supply, etc. This can be obtained by Low-flow Frequency Analysis (LFA) involving fitting probability distribution to the series of Annual Minimum d-day Average Flow (AMdAF) for different duration of 'd' such as 7-, 10-, 14- and 30-days, that is derived from the daily stream flow data series. This paper presents a study on LFA for Haora river at Haora gauging site adopting 2-parameter Log Normal (LN2) and Weibull distributions. Parameters of the distributions are determined by method of moments, Maximum Likelihood Method (MLM) and L-Moments, and are used for estimation of low-flow. The adequacy of fitting LN2 and WB2 distributions to the AMdAF data series is evaluated by non-parametric Goodness-of-Fit (GoF) test, say, Kolmogorov-Smirnov. Model performance analysis using Model Performance Indicators (MPIs) viz., correlation coefficient and root mean squared error are applied for the selection of best suitable distribution for estimation of low-flow. The GoF test results and MPIs values presents the LN2 (using MLM) distribution is better suited for LFA for Haora gauging site.

Keywords: *Correlation Coefficient, Kolmogorov-Smirnov, Log Normal, Low-flow, L-Moments, Method of Moments, Maximum Likelihood Method, Root Mean Squared Error, Weibull*

INTRODUCTION

Low-flow analysis is an important aspect for water quality management, reservoir storage design, determining minimum release policy and safe surface water withdrawals. Low-flow is seasonal phenomenon and an integral component of flow regime of any river [1]. Hydrological literature described that there are many inter-linking natural factors, which contributes to low-flow. This includes direct river withdrawals for human activity and artificial afforestation in the catchment. Numbers of indices such as mean annual runoff, mean daily flow, median flow, Annual Minimum d-day Average Flow (AMdAF), absolute minimum flow are widely used to characterize the low-flow. Among these, AMdAF is generally adopted procedure for characterizing low-flow in a stream, which satisfy the condition afore-mentioned is by averaging the flow using moving average method for 'd' consecutive days such as 7-, 10-, 14- and 30- days. Values of 'd' larger than unity help in diminishing the effect of fluctuations resulting from minor river regulations. An associated, annual event based, low-flow statistic q(d,T) gives low-flow estimate, which is defined as the AMdAF that is expected to be occurred once in T-year return period [2]. Generally, the available stream flow data are insufficient to conduct an accurate analysis of an extreme low-flow event. Therefore, for improving the accuracy of estimated low-flow, number of probability distributions is applied [3]. After extracting the AMdAF series from daily stream flow data series, probability distributions are adopted in Low-flow Frequency Analysis (LFA) to estimate the value of q(d,T).

Research reports iterated that the Normal, Gumbel, 2-parameter Log Normal (LN2), Pearson Type-3, Log Pearson Type-3 (LP3) and 2-parameter Weibull (WB2) distributions are commonly available for LFA [4]. Ahn et al. [5] applied Power and SMEMAX (Small, MEdium and MAXimum) transformation, Weibull, LP3 and LN2 distributions to estimate the 7-day and 30-day low-flows for different return periods at four gauged points of the Ansung stream in Korea. Bowers et al. [6] analyzed the seasonal river flow data and found both power law and LN2 distributions are relevant to dry seasons. They also found that the river flow data in wet seasons are typically better-fitted using LN2 distribution than power law distribution. Randall and Freehafer [7] applied regression method to study on low-flow statistics at ungauged sites in the Lower Hudson River Basin, New York. They found that the logarithmic transformation yielded less accurate equations inconsistent with some conceptualized relationships. In the present study, LN2 and WB2 distributions are adopted in LFA for estimation of q(d,T).

Generally, Method of Moments (MoM) is widely applied for determination of parameters of the probability distributions [8]. Since MoM estimates are usually inferior in quality especially for distributions with three or more number of parameters because higher order moments are more likely to be highly biased in relative small samples [9]. Moreover, the studies carried by various researchers indicated that the estimated parameters of distributions fitted by the MoM are often less accurate than those obtained by other parameter estimation procedures viz., Maximum Likelihood Method (MLM) and L-Moments (LMO). A non-parametric Goodness-of-Fit (GoF) test, say, Kolmogorov-Smirnov (KS) is used for checking the adequacy of fitting of LN2 and WB2 distributions adopted in LFA. Model Performance Indicators (MPIs) viz., Correlation Coefficient (CC) and Root Mean Squared Error (RMSE) are used for the selection of a suitable probability distribution for estimation of q(d,T). This paper presents the applicability of MoM, MLM and LMO of LN2 and WB2 distributions adopted in LFA for river Haora at Haora gauging site.

METHODOLOGY

Analysis of low-flow of a stream pre-supposes that: (i) no significant withdrawals and diversions from the location points are in operation and (ii) flow in a river or stream to be natural. The AMdAF series for different duration of ‘d’ such as 7-, 10-, 14-, and 30-days are subsequently obtained from the daily stream flow data series. These values are used to determine the parameters of LN2 and WB2 distributions for estimation of q (d,T).

Log Normal Distribution

The Probability Distribution Function (PDF) and Cumulative Distribution Function (CDF) of LN2 are given by:

$$f(q)=\frac{1}{\beta q\sqrt{2\pi}}e^{-\left(\frac{(\ln(q)-\alpha)^2}{2\beta^2}\right)},\ q>0,\ \beta>0 \qquad \dots (1)$$

$$F(q)=\varphi\left(\frac{\ln(q)-\alpha}{\beta}\right) \qquad \dots (2)$$

Where, q is the variable (i.e., low-flow), α is the scale parameter, β is the shape parameter, f(q) is the PDF of q and F(q) is the CDF of q [10]. The estimators of scale $(\hat{\alpha})$ and shape $(\hat{\beta})$ parameters are determined by MoM, MLM and LMO; and are used to estimate q (d,T) for a given return period (T) and duration ‘d’ from Eq. (3), which is given as below:

$$q(d,T)=\exp(\hat{\alpha}+K_T\hat{\beta}) \qquad \dots (3)$$

Where, $\phi(\dots)$ is the CDF of the standard normal distribution. The frequency factor (K_T) for a return period (T) is computed from Eq. (4), and given by:

$$K_T=4.91\left((1/T)^{0.14}-(1-(1/T))^{0.14}\right) \qquad \dots (4)$$

The procedures involved in determining the estimators of the parameters of LN2 by MoM, MLM and LMO are briefly described in the following sections:

MoM of LN2

$$\hat{\alpha}=\ln\left(\frac{1}{N}\sum_{i=1}^{N}q_i\right)-\frac{1}{2}\ln\left(\frac{1}{N}\sum_{i=1}^{N}q_i^2\right)+\frac{1}{2}\ln\left(\frac{1}{N}\sum_{i=1}^{N}q_i\right)^2 \qquad \dots (5)$$

$$\hat{\beta}=\left[\ln\left(\frac{1}{N}\sum_{i=1}^{N}q_i^2\right)-2\ln\left(\frac{1}{N}\sum_{i=1}^{N}q_i\right)\right]^{1/2} \qquad \dots (6)$$

Here, q_i is the observed low-flow (q) for i^{th} sample.

MLM of LN2

$$\hat{\alpha}=\frac{1}{N}\sum_{i=1}^{N}\ln(q_i)\ \text{and} \qquad \dots (7)$$

$$\hat{\beta}=\left(\frac{1}{N}\sum_{i=1}^{N}(\ln(q_i)-\bar{q})^2\right)^{1/2} \qquad \dots (8)$$

Where, $\bar{q}$ is the average of observed low-flows.

LMO of LN2

The LMO estimators of the parameters of LN2 [11] are determined by:

$$\hat{\alpha}=\lambda_1 \text{and}\quad \lambda_2=\hat{\beta}/\sqrt{\pi} \qquad \dots (9)$$

$$\lambda_1=b_0=\frac{1}{N}\sum_{i=1}^{N}\ln(q_i) \qquad \dots (10)$$

$$\lambda_2=2b_1-b_0 \qquad \dots (11)$$

$$b_1=\frac{1}{N(N-1)}\sum_{i=2}^{N}(i-1)\ln(q_i) \qquad \dots (12)$$

Here, λ_1 and λ_2 are first and second LMOs, N is the sample size and $\ln(q_i)$ is the logarithmic value of q_i (viz., AMdAF for different duration of ‘d’ such as 7-, 10-, 14- and 30-days) arranged in ascending order (i.e., $q_1<q_2<.....<q_N$).

Weibull Distribution

The PDF and CDF of WB2 distribution [12] is given by:

$$f(q)=\frac{\beta}{\alpha}\left(\frac{q}{\alpha}\right)^{\beta-1}e^{-\left(\frac{q}{\alpha}\right)^{\beta}} \quad \ldots (13)$$

$$F(q)=1-e^{-\left(\frac{q}{\alpha}\right)^{\beta}},q>0,\alpha>0,\ \beta>0 \quad \ldots (14)$$

The parameters are determined by MoM, MLM and LMO; and are used to estimate q(d,T) from Eq. (15), and given by:

$$q(d,T)=\hat{\alpha}(-\ln(1-(1/T)))^{(1/\hat{\beta})} \quad \ldots (15)$$

The procedures adopted in determining the estimators of the parameters of WB2 by MoM, MLM and LMO are briefly described in the following sections:

MoM of WB2

$$\mu=\hat{\alpha}\Gamma\left(1+\left(1/\hat{\beta}\right)\right) \quad \ldots (16)$$

$$\sigma=\hat{\alpha}\left[\Gamma\left(1+\left(2/\hat{\beta}\right)\right)-\Gamma^2\left(1+\left(1/\hat{\beta}\right)\right)\right]^{1/2} \quad \ldots (17)$$

where, μ and σ are the average and standard deviation of the observed AMdAF for different duration of ‘d’ [13].

MLM of WB2

The MLM estimator of the shape parameter $(\hat{\beta})$ of WB2 [14] is obtained by using Newton-Raphson iteration, which is given as below:

$$\frac{1}{\hat{\beta}}=\frac{\sum_{i=1}^{N} q_i^{\hat{\beta}}\ln(q_i)}{\sum_{i=1}^{N} q_i^{\hat{\beta}}}-\frac{1}{N}\sum_{i=1}^{N}\ln(q_i) \quad \ldots (18)$$

The estimator of the scale parameter $(\hat{\alpha})$ is subsequently obtained from the following equation:

$$\hat{\alpha}=\left(\frac{1}{N}\sum_{i=1}^{N} q_i^{\hat{\beta}}\right)^{1/\hat{\beta}} \quad \ldots (19)$$

LMO of WB2

The LMO estimators [15] of the parameters of WB2 can be computed by solving the Eqs. (20) and (21), which are as follows:

$$\lambda_1=\hat{\alpha}\Gamma\left(1+\left(1/\hat{\beta}\right)\right) \quad \ldots (20)$$

$$\lambda_2=\hat{\alpha}\left(1-2^{-(1/\hat{\beta})}\right)\Gamma\left(1+\left(1/\hat{\beta}\right)\right) \quad \ldots (21)$$

Goodness-of-Fit Tests

A non-parametric GoF test, say, Kolmogorov-Smirnov (KS) is applied for checking the adequacy of fitting of LN2 and WB2 distributions adopted in LFA. The theoretical description of KS test statistic [16] is given as below:

$$KS=\mathrm{Max}\sum_{i=1}^{N}\left|F_e(q_i)-F_D(q_i)\right| \quad \ldots (22)$$

Here, $F_e(q_i)$ is the empirical CDF of q_i using Weibull plotting position formula for i=1,2,3,…,N with $q_1<q_2<....<q_N$, $F_D(q_i)$ is the derived CDF of q_i using LN2 and WB2 distributions, and N is the sample size. The theoretical value of KS test statistic for different sample size (N) for different significance level is available in the technical note on ‘Goodness-of-Fit Tests for Statistical Distributions’ [17].

Test criteria: If the computed value of GoF test statistic given by the distribution (or method) is less than that of its theoretical value at the desired significance level then the selected distribution (or method) is acceptable for LFA.

Model Performance Analysis

The selection of a suitable probability distribution for estimation of q(d,T) is carried out through model performance analysis using MPIs (viz., CC and RMSE). The theoretical expressions of CC and RMSE [18] are as follows:

$$CC=\frac{\sum_{i=1}^{N}\left(q_i(o)-\overline{q(o)}\right)\left(q_i(e)-\overline{q(e)}\right)}{\sqrt{\sum_{i=1}^{N}\left(q_i(o)-\overline{q(o)}\right)^2\sum_{i=1}^{N}\left(q_i(e)-\overline{q(e)}\right)^2}} \quad \ldots (23)$$

$$RMSE=\left(\frac{1}{N}\sum_{i=1}^{N}\left(q_i(o)-q_i(e)\right)^2\right)^{1/2} \quad \ldots (24)$$

where, $q_i(o)$ and $q_i(e)$ are the observed and estimated low-flows respectively for ith sample, $\overline{q(o)}$ is the average of observed low-flows and $\overline{q(e)}$ is the average of estimated low-flows.

Selection criteria: The distribution with high CC (say, CC>0.9) and minimum RMSE is identified as better-suited distribution for estimation of low-flow.

Application

In the present study, a study on estimation of q(d,T) using the AMdAF series for different duration of ‘d’ such as 7-, 10-, 14- and 30-days by adopting LN2 and WB2 distributions for river Haora at Haora gauging site is carried out. The Haora river is flowing through low undulating topography with an elevation vary between 6 m to 201 m. After originating from the western flank of the Baramura range, the river flows through hilly tracts for a distance of 6.6 km and debouches onto the foothill zone near Chandrasadhubari (83 m). The total length of the Haora river in the state is about 61.2 km of which 52 km is flowing within Indian Territory. The catchment area of river Haora is about 457.92 km^2. The daily stream flow data observed at Haora gauging site for the period 1990 to 2009 is used. Figure 1 presents the location map of the study area.

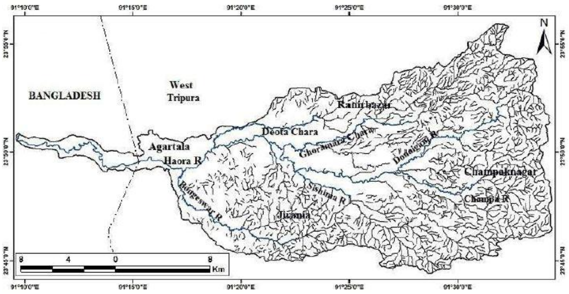

Figure1. Location map of the study area

Results and Discussions

The topography of the Haora river basin indicates the stream flow response varies during non-monsoon months owing to the occurrence of non-monsoon rainfall and flow regulations by the upstream reservoir. The examination of stream flow data observed at Haora gauging site indicates that the river was perennial. The AMdAF series for different duration of ‘d’ such as 7-, 10-, 14-, and 30-days was extracted from the daily stream flow data series, and are used for LFA. Table 1 presents the descriptive statistics (viz., average, Standard Deviation (SD), Coefficient of Skewness (CS) and Coefficient of Kurtosis (CK)) of the AMdAF data series.

Table1. Descriptive statistics of AMdAF series

AMdAF series	Average (m^3/s)	SD (m^3/s)	CS	CK
d=7	1.625 (0.422)	0.600 (0.375)	1.030 (-0.487)	1.573 (2.044)
d=10	1.731 (0.483)	0.650 (0.380)	1.016 (-0.408)	1.469 (1.583)
d=14	1.824 (0.532)	0.690 (0.393)	0.860 (-0.545)	1.024 (1.545)
d=30	2.249 (0.746)	0.821 (0.375)	0.693 (-0.260)	0.573 (-0.145)
Figures given within the brackets indicates the descriptive statistics of the log-transformed data				

Estimation of q(d,T) using LN2 and WB2

In the present study, the estimators of the parameters of LN2 and WB2 probability distributions was determined by MoM, MLM and LMO for the AMdAF data series for different duration of ‘d’ such as 7-, 10-, 14- and 30-days. The parameters were used to estimate the q(d,T) for different duration ‘d’ and for different return period ‘T’; and the results are presented in Tables 2 to 5.

Table2. Estimated q(7,T) for different return periods using LN2 and WB2 distributions

Return period (year)	q(7,T) (m^3/s)					
	LN2			WB2		
	MoM	MLM	LMO	MoM	MLM	LMO
1.01	3.449	3.654	3.540	3.057	3.088	2.958
2	1.529	1.524	1.524	1.608	1.604	1.616
5	1.140	1.112	1.125	1.096	1.084	1.125
10	0.978	0.943	0.959	0.850	0.837	0.886
20	0.862	0.823	0.841	0.666	0.652	0.704
25	0.830	0.791	0.810	0.616	0.603	0.655
50	0.747	0.706	0.726	0.486	0.473	0.523
100	0.679	0.637	0.658	0.383	0.372	0.419

Table3. Estimated q(10,T) for different return periods using LN2 and WB2 distributions

Return period (year)	q(10,T) (m^3/s)					
	LN2			WB2		
	MoM	MLM	LMO	MoM	MLM	LMO
1.01	3.715	3.929	3.843	3.293	3.317	3.198
2	1.626	1.621	1.621	1.711	1.708	1.718
5	1.207	1.178	1.187	1.157	1.148	1.185
10	1.032	0.996	1.009	0.893	0.883	0.927
20	0.908	0.868	0.882	0.696	0.686	0.732
25	0.874	0.834	0.848	0.643	0.633	0.679
50	0.785	0.743	0.758	0.505	0.495	0.539
100	0.713	0.670	0.685	0.397	0.388	0.429

Table4. Estimated q(14,T) for different return periods using LN2 and WB2 distributions

Return period (year)	q(14,T) (m^3/s)					
	LN2			WB2		
	MoM	MLM	LMO	MoM	MLM	LMO
1.01	3.935	4.253	4.157	3.487	3.496	3.406
2	1.712	1.702	1.702	1.801	1.800	1.808
5	1.268	1.223	1.233	1.214	1.211	1.238
10	1.083	1.029	1.042	0.934	0.931	0.963
20	0.952	0.892	0.907	0.727	0.724	0.757
25	0.916	0.856	0.871	0.672	0.668	0.702
50	0.822	0.760	0.775	0.526	0.522	0.555
100	0.746	0.683	0.699	0.412	0.409	0.439

From LFA results, it is noted that the q(d,T) estimates for different duration of d such as 7-, 10- and 14-days obtained from WB2 (using MLM) distribution are lower than those values of LN2 (using MoM, MLM and LMO) and WB2 (using MoM and LMO) for return periods from 5-year to 100-year. The results also showed that estimated low-flows by WB2 are lower than those values of LN2 for return periods from 10-year to 100-year.

Table5. Estimated q(30,T) for different return periods using LN2 and WB2 distributions

Return period (year)	q(30,T) (m^3/s)					
	LN2			WB2		
	MoM	MLM	LMO	MoM	MLM	LMO
1.01	4.741	5.050	5.111	4.202	4.202	4.190
2	2.118	2.108	2.108	2.228	2.229	2.229
5	1.584	1.538	1.531	1.525	1.526	1.528
10	1.361	1.304	1.296	1.186	1.187	1.190
20	1.200	1.138	1.129	0.932	0.933	0.937
25	1.157	1.094	1.084	0.864	0.865	0.868
50	1.042	0.977	0.967	0.682	0.684	0.687
100	0.948	0.882	0.871	0.540	0.541	0.544

Low-Flow Frequency Curves (LFCs)

The estimated values of q(d,T) for different return periods from 1.01-year to 100-year for different duration of d viz., 7-, 10-, 14-, and 30-days were used to develop LFCs and presented in Figure 2. From LFCs, it can be seen that there is a line of agreement between the observed and estimated low-flows using LN2 when compared to WB2. Moreover, from LFCs, it is noted that the observed low-flows are nearer to the estimated low-flows using LN2 (MLM).

Analysis Based on GoF Test

GoF test values of KS test for different duration of 'd' viz., 7-, 10-, 14-, and 30-days were computed and are presented in Table 6.

Table6. Computed values of KS test statistic using LN2 and WB2 distributions

AMdAF series	Computed values of KS test statistic					
	LN2			WB2		
	MoM	MLM	LMO	MoM	MLM	LMO
d=7	0.105	0.102	0.100	0.140	0.139	0.140
d=10	0.100	0.098	0.108	0.147	0.146	0.148
d=14	0.096	0.112	0.118	0.152	0.152	0.149
d=30	0.102	0.085	0.090	0.104	0.104	0.105

From KS test results, it is noted that the computed values by LN2 and WB2 (using MoM, MLM and LMO) distributions are less than its theoretical value of 0.279 at 5% significance level, and at this level, both LN2 and WB2 distributions are acceptable for LFA.

Model Performance Analysis

In addition to GoF test, the performance of LN2 and WB2 distributions adopted in LFA using the AMdAF series for different duration of 'd' such as 7-, 10-, 14-, and 30-days was evaluated by MPIs. The MPIs values for LN2 and WB2 for the AMdAF data series were computed and are presented in Table 7. From these values, it is noted that the LN2 (using MoM, MLM and LMO) gave high CC and minimum RMSE when compared with the corresponding values of WB2 (using MoM, MLM and LMO) for the series of AMdAF for different duration of 'd'.

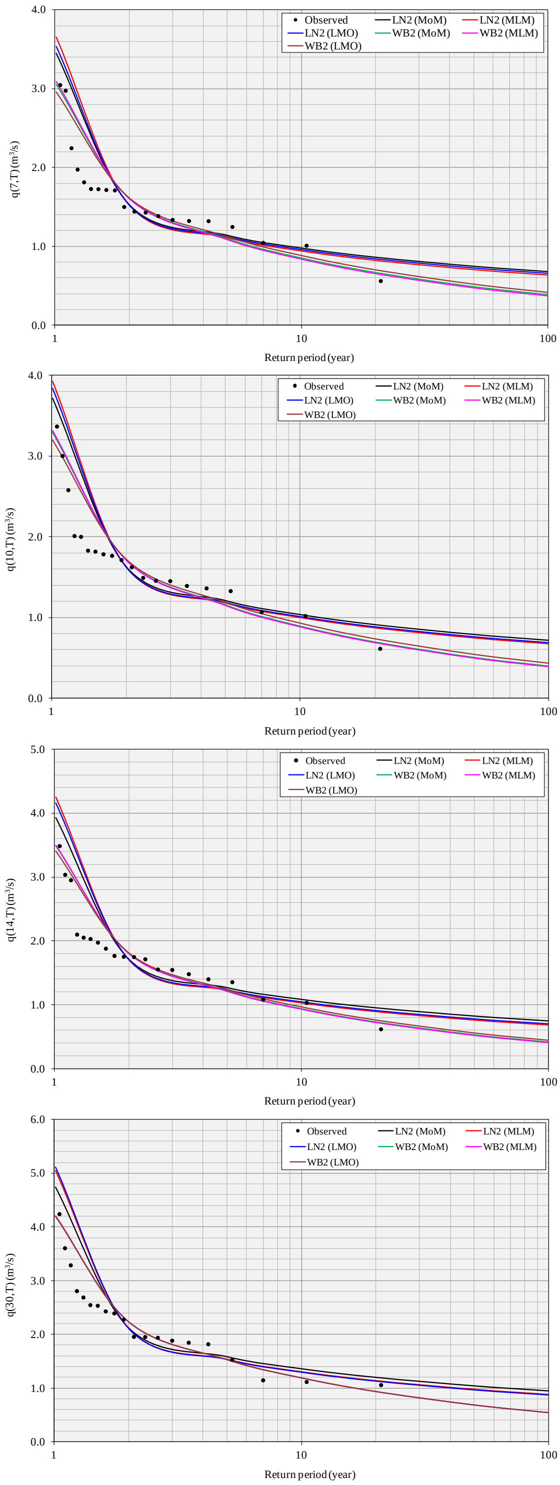

Figure2. Plots of estimated low-flows using LN2 and WB2 distributions with observed low-flows

Also, from Table 7, it is noted that RMSE given by LN2 (MLM) is minimum when compared with those values of LN2 (using MoM and LMO). The CC between observed and estimated low-flows using LN2 and WB2 distributions vary between 0.940 and 0.986. Based on GoF test values and MPI values, the study indicated that LN2 (using MLM) could be considered as an appropriate distribution for estimation of low-flow.

Table7. MPIs values given by LN2 and WB2 distributions for the AMdAF series

AMdAF series	CC						RMSE (m^3/s)					
	LN2			WB2			LN2			WB2		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
d=7	0.965	0.966	0.965	0.942	0.943	0.940	0.168	0.163	0.165	0.197	0.195	0.204
d=10	0.973	0.973	0.973	0.950	0.950	0.948	0.168	0.162	0.165	0.200	0.198	0.208
d=14	0.973	0.973	0.973	0.956	0.956	0.955	0.179	0.166	0.171	0.201	0.200	0.208
d=30	0.986	0.983	0.984	0.972	0.975	0.972	0.177	0.160	0.168	0.197	0.186	0.198

Conclusions

The paper presents the study carried out for estimation of q(d,T) for different duration of ‘d’ viz., 7-, 10-, 14- and 30-days through LFA by adopting LN2 and WB2 (using MoM, MLM and LMO) distributions for river Haora at Haora gauging site. From the results, the following conclusions were drawn from the study.

- KS test results confirmed the applicability of LN2 and WB2 distributions adopted in LFA.
- Low-flow estimates obtained from WB2 distribution are consistently lower than the corresponding values of LN2 for return periods from 10-year to 100-year.
- CC values given by LN2 and WB2 distributions indicated that there is a good correlation between the observed and estimated low-flows and these values vary between 0.940 and 0.986.
- The LFC curves showed that the estimated low-flows by LN2 (using MLM) distribution are nearer to the observed low-flows.
- Qualitative assessment (plots of LFA results) of the outcomes was weighed with RMSE values and accordingly LN2 (using MLM) distribution was found to be better suited amongst LN2 and WB2 distributions adopted in LFA for estimation of low-flow.

The study indicated that the estimated low-flows using LN2 (MLM) distribution could be useful to the stakeholders while deciding environmental flows and minimum water release policy and so on.

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